ARITHMETIC
IN
EPITOME:
OR, A
COMPENDIUM
OF ALL THE
RULES,
BOTH
VULGAR and DECIMAL.

WHEREIN
CLEAR and PLAIN DEMONSTRATIONS are deduced from the Principles of *Arithmetic* itſelf; without either Reference to *Euclid*, or Uſe of *Algebra*.

By W. WEBSTER, Writing-Maſter.

The TENTH EDITION,
Carefully Corrected, with ADDITIONS, by
ELLIS WEBSTER, *of the* Cuſtom-Houſe.

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BRITISH MUSEUM
27 SEP 57

To the RIGHT HONOURABLE

Sir *Robert Walpole*,

Knight of the Moſt Noble Order
of the Garter, Firſt Lord Commiſſioner
of the Treaſury, and Chancellor of
the Exchequer, *&c.*

SIR,

THOUGH the Diſtruſt ſo natural to a young Author, on his firſt Performance, did indeed diſcourage me from complying with my Inclination, in offering this attempt to your HONOR on its firſt Publication; yet, being a little encouraged by the favourable Reception its firſt Edition met with in the World, I did with all Humility preſume to lay the ſecond Impreſſion at your Feet; well knowing the Advantage it would appear with under the Patronage of a NAME ſo eminent for the Encouragecouragement of *Clerkſhip*, and of every other Part of uſeful *Literature*.

Upon the third Reviſal of this Work for the Preſs, I made ſuch Additions and Improvements, as have, I hope, given it ſomewhat a better Title to ſo great a Protection; but ſtill its Imperfections, I am very ſenſible, will be but too apparent to Your HONOR's Penetration: Yet that Goodneſs, of which I have already had ſo abundant Experience, gives me Hopes Your Honor will both excuſe what may remain amiſs, and forgive this renewed Preſumption of,

Your HONOR's

Moſt Obedient, and

Moſt Humble Servant,

William Webſter.

PREFACE.

THE Indulgence with which the former Editions of this Treatiſe have been received, has encouraged me to endeavour the making it ſtill more worthy the Acceptance of the Public ; not only by correcting the many Errors which had before eſcaped, but alſo by enlarging it with very conſiderable Additions : Having from the third Impreſſion added the *Theory* to the *Practic Part*, and endeavoured to inform the Reader's Judgment, as well as to aſſiſt his Memory.

And this I have attempted in a Manner very different from the ordinary Practice of Writers on this Subject, having never once quoted *Euclid*, nor perplexed the Learner with (to him unintelligible) *Algebraical* Demonſtrations.

ARITHMETIC and GEOMETRY are juſtly looked upon as the two main Pillars of the Mathematics, upon which the whole

Fabric is raiſed; but in comparing them, they are too often confounded: For, how great a Similitude ſoever there may be betwixt them, in ſome reſpects, yet, as their Subjects are different, ſo their Principles are certainly diſtinct, and each may be accounted for without References to the other. Such Compariſons may indeed be uſeful by way of Illuſtration, and give great Satisfaction to thoſe who have a competent Knowledge in both; but Concluſions from *Geometrical* Demonſtrations are beyond the reach of the young Arithmetician; and the Theory of *Lines* and *Diagrams*, as they are commonly made uſe of, can only ſerve to bewilder and confound him.

Algebra, indeed, is not properly a Science diſtinct from Arithmetic, but is only a different Method of Computation, performed by ſubſtituting *Letters* in the Place of *Figures*, and expreſſing the ſeveral Parts of the Operation by *Symbolical Characters*. 'Tis true, great Diſcoveries have been made in Numbers by this Means, and Arithmetic owes many of its Improvements to this admirable Invention: Beſides, it furniſhes us with a ſhort Way of arguing, and brings the Proofs of a Propoſition into a little Compaſs. But however uſeful it may be to thoſe who underſtand it, it is certainly an unintelligible Jargon to the mere Numeriſt, and can give him very little ſatisfaction when laid before him as a Demonſtration.

Our knowledge in ſcience ſhould riſe gradually from one ſtep to another, and our references ſhould be always backward to what went before; but as the uſeful parts of Arithmetic have been commonly taught by practical Rules, which are uſually taken upon content, ſo little exactneſs is generally obſerved in the order of their delivery, as to their dependance upon each other: The reaſon of which is plain; for as all Affairs do not require the whole knowledge of Numbers, and as moſt common Buſineſs may be done without much Skill in *Fractions*; ſo it is not only common, but reaſonable enough, to carry the Learner thro' whatever may be performed by *Whole Numbers*, before he is entered upon the greater difficulties of *Fractional Parts.* However, the gradation mentioned ſhould be obſerved as much as poſſible, eſpecially in the Buſineſs of Demonſtration; and the Maxims referred to for explanation ſhould never be of a more abſtruſe nature than the thing to be explained.

I have therefore endeavoured to give the inquiſitive Reader, who would underſtand the reaſon of things, all the ſatisfaction I can ; and have attempted to account for the Principles of Arithmetic from ſuch ſelf-evident Propoſitions, and natural conſiderations, as flow moſt immediately from Numbers themſelves, and fall within the Comprehenſion of ſuch who are the common Conſulters of *Arithmetical Tracts* ; and who, it may be preſumed, are generally unacquainted with the more abſtruſe parts of the Mathematics.

My firſt Deſign in this Treatiſe was, according to its Title, to epitomize the Art : and to reduce to a *Pocket-ſize* what has ſometimes been ſwell'd to a *Folio*. Indeed what I firſt publiſhed was no more than a practical Memorandum, and, conſequently conſiſted only of Rules and Examples, with few or no Explications ; but tho' the Inlargements are now very conſiderable, I have not yet gone beyond my Deſign ; it is ſtill a Compendium, and far ſhort in bulk of other Treatiſes on the ſubject. I may therefore now recommend it to the ingenious *Clerks* and *Accomptants* of the ſeveral Offices of *Great Britain*, (for whoſe Uſe eſpecially it was at firſt deſign'd) and to ſuch other *Gentlemen* whoſe Buſineſs requires the practice of Arithmetic, with more confidence than before ; it being very little increas'd in Size, tho' very much improv'd in Precept.

As

PREFACE.

As I ſuppoſe my Reader acquainted with the manner of performing the common Rules of *Addition*, *Subtraction*, *Multiplication*, and *Diviſion*; ſo I have made the work ſhort, by omitting ſuch needleſs Inſtructions: but the *Reaſons* of thoſe Operations (being the Fundamentals of Arithmetic) I thought neceſſary to lay down. The *Rule of Three* I have alſo endeavoured ſo to explain, that the Application of *Proportion*, as it runs thro' almoſt all other Rules, may be fully underſtood.

And for thoſe Practices of Arithmetic which will admit of no other than an *Algebraical* Demonſtration, being generally Parts more curious than uſeful, ſuch as are ſome propoſitions in *Progreſſion* and the *Double Rule of Falſe*; I have thought it ſufficient to give the practical Rules for their Performance, without their Demonſtration; ſince thoſe who are Proficients in *Algebra* cannot be thought to want ſuch Explanation; and thoſe who are not, cannot be ſuppoſed to underſtand it.

I have indeed ventured to account for the Extraction of the Roots, in a manner which ſeems ſomething contradictory to my profeſs'd Deſign: but when it is conſidered, that the Subject of thoſe Operations is entirely *geometrical*, though the Operations themſelves are *numerically* performed, it will be found that there is no other rational Method of Demonſtration, but that of *Lines*, which, however, I hope, I have ſo laid down, as to be clear and intelligible to every Reader of common ſenſe, who but underſtands the difference betwixt a *Line*, *a Surface*, and a *Solid*.

As to the Diſpoſition of the *Chapters*, they follow much in the ſame order as they are generally taught in *Schools:* For tho' I am ſenſible, that the Rules of *Practice* are not thoroughly to be underſtood without ſome knowledge of *Vulgar Fractions*, yet I have choſen to place Fractions in the laſt Chapter of *Vulgar Arithmetic*, becauſe (as I before obſerved) moſt common Buſineſs may be done by whole Numbers. So the Speculative Rules of *Progreſſion*, or *Arithmetical* and *Geometrical Proportion*, which, in a juſt *Theoretic* way of treating the ſubject of Numbers, ought to precede the *Rule of Three*, I have thought better to place juſt before *Vulgar Fractions*: however, *Progreſſion* and *Vulgar Fractions*, being each diſtinct Chapters, the Reader may look into them whenever he thinks it moſt proper.

PREFACE.

The ſecond Part of this Treatiſe, which contains the Doctrine of *Decimal Fractions*, &c. is a kind of Arithmetic peculiarly, as it were, adapted to the concerns of *Gentlemen:* I have therefore been more large than ordinary upon that Subject, and have run over the ſeveral Rules again, to ſhew its particular uſe and application. In the Chapter of *Intereſt* and *Rebate*, its excellency will moſt appear, every Calculation of that kind being performed to great advantage by Decimal Numbers. Under that head will likewiſe be found ſeveral uſeful *Tables* (which, that they might be perfectly correct, are exactly engraved on Copper) for the ready diſcovering the amount, and preſent worth both of ſingle Sums, and of Annuities. And as the Intereſt of Money is now at a lower Rate than when this Treatiſe was firſt publiſhed, I have in this Impreſſion added Tables, at 3 and 4 *per Cent.* to thoſe before publiſhed at 5 and 6. There are, beſides theſe, ſome other neceſſary *Tables*, interſperſed through this part of Arithmetic (which are alſo with the ſame exactneſs, printed from Copper-Plates); as thoſe for the reduction of the known parts of Coin, Time, Weight, and Meaſure, into decimal Parts of their proper *Integers*; and alſo that moſt uſeful *Table*

ble, which ſhews the number of Days from any day in any Month to the ſame day in any other. The *Contents* will ſhew where to find any of theſe things; and in the Work itſelf are ſufficient Explanations both of their formation and uſe. I ſhall therefore add nothing farther by way of Preface, but ſubmit the whole Performance to the judgment and candour of the Public.

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VULGAR ARITHMETIC,

IN

Whole Numbers and Fractions.

AS it is inconſiſtent with the intended brevity of this Treatiſe, to enter upon a nice enquiry after the firſt Inventors of Arithmetic, or by what degrees it has been raiſed to its preſent perfection; ſo neither is it my deſign, in the following pages, to initiate abſolute Beginners; but ſuppoſing my reader acquainted, at leaſt, with the methods of Adding, Subtracting, Multiplying, and Dividing, (tho' perhaps ignorant of the reaſons thereof) I ſhall lay before him, in as few words as poſſible, the reaſons of thoſe Fundamentals, and their Application throughout all the varieties of practice.

Arithmetic (which is juſtly defined *The Art of Reckoning*) has for its ſubject, Number; and teaches us to give proper anſwers to all ſuch queſtions as demand, *How many?* For the more ready performing of which, the ten following Characters, or Figures, have been invented and agreed upon by the greateſt part of the known World, *viz.*

1 2 3 4 5 6 7 8 9 0

and by the various combinations and repetitions of theſe ten Characters, may every Number, how great ſoever, be eaſily and expeditiouſly expreſſed.

Thus much in general: I now proceed to particulars.

CHAP. I.

Of the five firſt and fundamental Rules, viz. *Numeration, Addition, Subtraction, Multiplication, and Diviſion.*

NUMERATION, or NOTATION.

BY Numeration we learn the different value of Figures, by their different places; and, of conſequence, to read or write any Sum, or Number.

The TABLE.

9	Units.	1
90	Tens.	12
900	Hundreds.	123
9000	Thouſands.	1234
90000	X Thouſands.	12345
900000	C Thouſands.	123456
9000000	Millions.	1234567
90000000	X Millions.	12345678
900000000	C Millions.	123456789

From this Table may be obſerved:

1. The names of the ſeveral places, *viz* Units, Tens, Hundreds, *&c.* which proceed increaſing by a tenfold proportion) from the right-hand to the left.

2. That every Figure hath two values; one in itſelf; the other from the place it ſtands in. Thus, on the left-ſide of the Table, the figure 9 in the upper line, ſtanding in the unit's place, is only nine; but in the

ſecond line, being removed into the place of Tens, becomes ninety; and in the third line is nine hundred, &c.

3. That tho' a cypher is nothing in itſelf, yet it gives value to other figures, by removing them into higher places.

All which being very obvious, I proceed to the next Rule,

ADDITION.

BY Addition we find the whole, or total, of two or more parts, or ſums.

In ſetting down the numbers to be added, care muſt be taken to place every figure in its proper column; that is, Units under Units, Tens under Tens, &c. Then will the reaſon of the work (the manner of which I ſuppoſe my reader well acquainted with) appear very evident from this undeniable Maxim, *viz.* *That the Whole is equal to all its Parts.* And the method of ſetting down the total may eaſily be accounted for from the nature of Numeration, which explains the different value of places, as they proceed from the right to the left hand: For as 9 is the greateſt ſimple character, or figure; ſo every number exceeding 9, being compound, muſt require more places than one to expreſs it. Thus the number 10 can no otherwiſe be expreſſed in figures, but by removing the figure 1 into the place of Tens, which is done by ſupplying the Unit's place with a cypher: And as it is the ſame with every other column (Ten being ſtill the proportion of increaſe) conſequently, when the ſum of any column amounts to 10, or more, the units exceeding, if there be any, or a cypher, if none, muſt be ſet under ſuch column; and the ten, or tens, of the amount, carried on as ſo many units, to the next column on the left.

What

The cyphers are only put down to shew the true distance of the first figure in each particular product from the units place

What is here obſerved, as to carrying the Tens (the proportion of increaſe) from one column to another in Integers, may be as juſtly applied to the numbers we ſtop at, in adding ſums of different Denominations.

For your greater eaſe in caſting up of Money, learn the following Table.

PENCE-TABLE.

Pence.		*s.*	*d.*	*Pence.*		*s.*	*d.*
20		1	8	80		6	8
30		2	6	90		7	6
40	is	3	4	100	is	8	4
50		4	2	110		9	2
60		5	0	120		10	0
70		5	10				

Examples in whole Numbers, and Money.

℔.	*Yards.*	*l.*	*s.*	*d.*	
756	1325	735	18	09 ½	
132	4532	423	10	10 ½	
458	7345	784	12	05 ¾	
736	1298	297	08	04	
857	8473	542	11	11 ¼	
241	5249	298	14	07 ¾	
3180	28222	3082	17	00 ¾	*Total.*

Examples in Avoirdupoiſe *and* Troy Weights.

AVOIRDUPOISE.

Tons.	*C.*	*q.*	℔.	*oz.*	*drams.*	
753	19	3	27	15	15	
347	08	1	17	10	06	
283	11	0	12	03	10	
549	05	2	13	09	07	
251	13	0	15	03	11	
2185	18	1	02	11	01	*Total.*

TROY-WEIGHT.

℔.	*oz.*	*dwts.*	*grs.*	
327	11	19	23	
429	10	17	19	
274	08	13	09	
478	10	12	05	
326	09	18	11	
1838	04	01	19	*Total.*
1510	04	01	20	*Tot. without the Top-line.*
1838	04	01	19	*Proof.*

To know how many Drams make an Ounce, or Grains a Penny-weight, &c. ſee the following Tables, Page 18.

Proof of Addition.

The proof of this Rule is uſually by a ſecond addition, without the top-line; which ſecond ſum, if, when added to the uppermoſt line, it makes the firſt total, the work is ſuppoſed right. *See the laſt Example.*

But it is as good, if not a Better Proof, to produce the ſame total, by adding your columns both up and down.

SUBTRACTION.

SUbtraction teaches us, by taking a leſſer number from a greater, to find the Remainder.

The reaſon of this Rule is evident from the ſame principles as Addition; Subtraction being but the reverſe thereof: And the number borrowed in any column (like what we ſtopt at in Addition) being always ſo many as would make 1 in the next.

Examples in Integers and Money.

	Yards.
	7146325
	1483972
Rem.	5662353
Proof.	7146325

	l.	*s.*	*d.*
Lent	812	13	08¼
Paid	190	19	10½
Rem.	621	13	09¾
Proof	812	13	08¼

	l.	*s.*	*d.*
Borrowed at ſeveral Times.	429	11	08½
	212	17	10
	356	17	05¾
Borrowed in all	999	07	00¼
Paid	519	18	09¾
Rem.	479	08	02½
Proof.	999	07	00¼

AVOIR-

AVOIRDUPOISE.

Tons.	*C.*	*q.*	*lb.*	*oz.*	*Drams.*	
92	10	0	07	03	13	
14	13	3	11	12	05	
77	16	0	23	07	08	*Rem.*
92	10	0	07	03	13	*Proof.*

TROY-WEIGHT.

	℔.	*oz.*	*dwts.*	*grs.*	
From	672	10	05	09	
Take	139	11	05	21	
	532	10	19	12	*Rem.*
	672	10	05	09	*Proof.*

Proof of Subtraction.

Sums in this Rule are eaſily proved, by adding their remainders to their leſſer numbers; which (if right) will make the greater.

MULTIPLICATION.

MUltiplication (which is the fourth Rule) ſerves in-ſtead of many Additions; the product of a Multiplication being only the repetition of the Multiplicand ſo many times as there are units in the Multiplier.

Note. The ready performance of this and the next Rule, intirely depends upon the perfect knowledge of the following Table.

The TABLE.

3 times	3	9
	4	12
	5	15
	6	18
	7	21
	8	24
	9	27
4times	4	16
	5	20
	6	24
	7	28
	8	32
	9	36
5 times	5	25
	6	30
	7	35
	8	40
	9	45
6 times	6	36
	7	42
	8	48
	9	54
7 times	7	49
	8	56
	9	63
8 times	8	64
	9	72
9 times	9	81
12times	2	24
	3	36
	4	48
	5	60
	6	72
	7	84
	8	96
	9	108
	10	120
	11	132
	12	144

Three terms are uſed in Multiplication, *viz. Multiplicand*, *Multiplier* and *Product*.

The following Example ſhews to which line each term belongs.

EXAM-

EXAMPLE.

```
       7563254138 Multiplicand.
         23456789 Multiplier.
       ----------
      68069287242
     60506033104
    52942778966              1
   45379524828             8+8 Proof.
  37816270690                1
 30253016552
22689762414
15126508276
------------------
177409656468442882 Product.
------------------
```

That Multiplication ſerves inſtead of many Additions, and conſequently, that the truth of its operation depends upon the ſame Reaſons, is eaſily proved: For ſuppoſe it were required to know the ſum of four ſevens; by Addition, the work will ſtand thus:

```
 7
 7
 7
 7
--
28
```

7
4
20

1
7 4
1

Too tedious a method for practice, being diſcoverable at once, by the Table of Multiplication to be 28.

Nothing therefore remains to be accounted for in this Rule, but the reaſon of placing every following particular product a place nearer to the left-hand than the product foregoing: But regard being ſtill had to the nature of Numeration, it will plainly appear, that as we begin with the right-hand figure of the Multiplier, or Unit's place; ſo the ſecond figure ſtanding in the place of Tens, the product thence ariſing, muſt, of con-

see p. 14. marks.

conſequence, be ten times greater than the ſame figure, ſtanding in the place of Units, would have produced; and ſo on, applying the ſame conſideration to the produce of all the other figures in the Multiplier.

Proofs of Multiplication.

Multiplication is uſually thus proved. Caſt out the *nines* from the Multiplicand and Multiplier, and place the remainders on the right and left ſides of a croſs, thus made +. Theſe two figures multiplied together, muſt have the *nines* caſt out of their product, and the remainder placed at top: Then caſting the *nines* alſo out of the product of your Multiplication, place its remainder at the bottom; which if it agrees with the figure at the top, the work is ſuppoſed right. *See the Example.*

A more certain Proof.

A Multiplication ſum is then right, when the product, divided by the Multiplier, quotes the Multiplicand; or, divided by the Multiplicand, quotes the Multiplier.

ABBREVIATIONS.

(*1ſt.*) When either your Multiplicand, or Multiplier, or both, have one or more cyphers to the right-hand, only multiply by the ſignificant figures, and ſet on the right-hand of the product ſo many cyphers as were in both the Multiplicand and Multiplier.

EXAMPLES.

$$\begin{array}{r} 527000 \\ 4000 \\ \hline 2108000000 \end{array} \qquad \begin{array}{r} 7583 \\ 7000 \\ \hline 53081000 \end{array}$$

(*2dly.*) Therefore any number may be multiplied by 10, 100, 1000, &c. only by placing on the right-hand

as multipⁿ. is continued addition, & 9 is the greatest simple figure (i.e. that takes up but one place. v. p. 3) the remr. being still carried to the left place, or next figure on left, it is proved by casting out the 9s & setting down the remainders. As addition is worked by 9s, no other figure would prove, & no other figure leaves a remainder for the next decimal place.

of it, one, two, three, or more cyphers: Thus 7295 multiplied by 10, is 72950; by 100, is 729500, &c.

(3*dly.*) To multiply any number by 5, add a cypher to it, and halve it. Or by 15, the ſame; and add both lines together.

EXAMPLES.

5334 *multiplied*

by 5 —— 53340

is —— 26670

by 15 —— 53340
26670

is —— 80010

(4*thly.*) Any number may be multiplied by 11, 111, or 112, &c. as in the following Examples.

EXAMPLES.

7136 *multiplied by* 11
7136

78496

by 111 *thus:* 7136
7136
7136

792096

And by 112, *thus:* 7136
7136
7136
7136

799232

Or thus: 7136
112

85632
7136

799232

We now proceed to our fifth Rule.

remainder
If the figures at the top of the cross, i.e. the rem.rs of the multip.d & multiplier answer to the figures or remainder of the product at the bottom the sum is supposed to be proved. If it be right it can not appear wrong as those products must be equal; but the remainder of unequals may be equal; e.g. 4 would be the remainder of 36 as well as of 28 & the sum be wrong.

DIVI-

DIVISION.

BY Diviſion (which is the reverſe of Multiplication) we find how often one number is contained in another.

In this rule there are alſo four terms, the *Diviſor*, *Dividend*, *Quotient*, and *Remainder*.

EXAMPLE.

```
Diviſor.    Dividend.     Quotient.
3456789)567895436783(164284
        --------
        22221653
        --------
         14809196
         --------
          9820407
          -------
          29068298
          --------
           14139863
           --------
             312707 Remainder.
```

As Multiplication ſerves inſtead of many Additions; ſo Diviſion ſupplies the place of many Subtractions; as may be thus made evident. Suppoſe it were required to divide 28 by 7, that is, to find how often 7 is contained in 28; by Subtraction the work will ſtand thus:

```
28
 7
--
21
 7
--
14
 7
--
 7
 7
--
```

Proof of the second method 32)42645(1332
units 32664
Tens 996
Rem. 21
42645
add

By which we find that 7 is 4 times contained in the number 28. But it may be diſcovered by the rules of Diviſion at one trial.

There are many methods of working this rule. The following Example is divided ſix ſeveral ways.

```
                     First.                  Second.
                     10(2                    10(2
                    1008(1                  1008(1
                    42645(1332           32)42645(1332
units in Div.   32222                   32664 units
Tens in div.      333                     996 Tens
                     Third.                  Fourth.
                     10(2                     21
                    1008(1                    —
                 32)42645(1332                 8
                                              10
                                             10
                                          32)42645(1332
                                              32
                                              —
                                              96
                                               96
                                                64
                     Fifth.                  Sixth.
                 32)42645(1332            32)42645(1332
                    32                        106
                    —                         —
                    106                       104
                     96                       —
                    —                          85
                    104                       —
                     96                        21
                    —
                     85
                     64
                    —
                     21
```

```
21 remains
08
10
10
32)42645(1332
   5 1
  1332
32)42645 | 1
   000   | 2
   11 .
      3/0
   5/ 3
```

```
      8
     10
    10
32)42645(1332
1st  32
2d    96
3d     96
4th     64
Rem.     21
       42645
add
```

```
      24(3 rem.                  or:  24(3
  5)793(158                        5)793(158
1st div:  5                          550
2d        25                         24
3d         40                          3 rem.
rem.        3.                     793
add                                add
          793
```

Note.

The numbers of Units & tens of divisor in each step of the divid^d (with remainder added) prove the sum

8569
multip:
8
72
480
4000
64000
68552

Note, The two laſt, called the *Italian* ways, are moſt generally uſed.

Mr. *Alingham* has, indeed, ſet down nine ways; but his three others are ſo very like the 1ſt, 2d, and 3d, of theſe, that they can ſcarce be called different.

ABBREVIATIONS.

(1*ſt*.) If there are any cyphers on the right-hand of your Diviſor, you may cut off ſo many cyphers, or figures, on the right-hand of your Dividend; but remember to bring them down (if figures) to the remainder.

EXAMPLE.

```
21|00)8645|29(411
      84
      —
       24
       21
       —
        35
        21
      ————
      1429
```

(2*dly*.) By the foregoing rule, you may obſerve, that to divide by 10, 100, 1000, &*c*. is only to cut ſo many figures from the right-hand of the Dividend, as there are cyphers in the Diviſor.

EXAMPLE.

1|000)43682|735)

So the Quotient is 43682, *the Remainder* 735.

(3*dly*.) When your Diviſor is 12, or conſiſts only of one ſingle figure, or can be reduced to one, by cutting off cyphers from its right hand, the work may be eaſily performed in one line, thus;

8)68552(8000 the first quotient figure
64000 (8 times 8000; subtract from dividend
(500 the 2d quotient figure
8) 4552 product is 4000, 8 times 500
subtr: 4000
(60 the 3d quotient figure
8) 552 product is 480, 8 times 60
subtr. 480
(9 the 4th quotient figure 9 times 8
8) 72
subtr. 72 9 because 9 stands in the place of unit
00 sum of all the quotients: 8000+500+60+9
or 8569

RULE.

RULE.

Drawing a line under the Dividend, ſet down under its firſt figure, how often the Diviſor is contained in it; what remains imagine placed before the next figure; and, conſidering how often your Diviſor is contained in the ſum it makes, ſet down the number underneath, as before; and ſo proceeding through all the figures, ſet down what remains at laſt, in the Place where your quotient uſed to ſtand.

EXAMPLES.

4)93645(1
23411

12)83675(11
6972

7|00)5635|15(
805

If you are to divide ſeveral Numbers by one common Diviſor (as in the calculating of Tables, &c.) that you may know exactly at once how often your Diviſor will go, in ſome convenient Corner make a Table of your Diviſor, by multiplying it ſeverally by all the nine Digits: Thus, ſuppoſe 562 your Diviſor:

562	1
1124	2
1686	3
2248	4
2810	5
3372	6
3934	7
4496	8
5058	9

Proofs of Diviſion.

(1*ſt.*) Multiplication and Diviſion mutually prove each other: For as if you divide the Product of a Multiplication by the Multiplier, the Quotient will be the Multiplicand; ſo, if you multiply the Quotient of a Diviſion by the Diviſor, (taking in the remainder) the product will be the Dividend.

(*2dly.*) Another proof of Diviſion is, by adding together thoſe lines in the following example, marked with Aſteriſms (being the particular products of the Diviſor, multiplied ſeverally by each figure in the Quotient, together with the remainder of the Diviſion) the total of which (if right) will be the Dividend.

(*3dly.*) Diviſion may alſo be proved as Multiplication, by a croſs, thus; caſting out the nines from the Diviſor and Quotient, place the remainders on its right and left ſides; then multiplying the two figures ſo placed together, and caſting the nines from the product, add what's left to the remainder of the Diviſion; and ſtill caſting out the nines, let the overplus be placed at the top; then alſo caſting the nines from the Dividend, ſet down the figure remaining at the bottom, which if it agrees with that at top, the work may be ſuppoſed right. See each proof in the following

EXAMPLE.

Place them,
736
736
5152
1472
664
863256

```
                736)863256(          1172
                    736*              736
                    ----             ----
                    1272             7032
                     736*            3516
                    ----             8204
     3              5365            ------
   7+2              5152*           862592
     3              ----               664 Remainder.
3d Proof.           2136            ------
                    1472*           863256 1ſt Proof.
                    ----
                     664*
                   ------
                   863256 2d Proof.
```

Before we proceed to *Reduction,* it will be proper to inſert the following Tables.

TABLES

a new way. Put the remainder of each dividend under the next figure to be brought down Quotient

```
        1172
736/ 863256
      863
      221
      852
```

1215
13) 15795 | 12

20 Inches
77)157|7 remainder 37 5 2/2 2 p 200/
3

TABLES *of* Engliſh *Coins, Weights, and Meaſures.*

Firſt, of COINS.

ACCOMPTS are kept in Pounds, Shillings, Pence, and Farthings, thus divided:

4 Farthings	make	1 Penny,	thus marked	*d.*
12 Pence		1 Shilling,		*s.*
20 Shillings		1 Pound,		*l.*

But the uſual Coins are,

			l.	*s.*	*d.*
Of Gold,	A Jacobus,	Value	1	5	0
	A Carolus,		1	3	0
	A Guinea,		1	1	0
	A ½ Guinea,		0	10	6
Of Silver,	A Crown,	Value	0	5	0
	A ½ Crown,		0	2	6
	The names of the reſt ſpeak their value, as a Shilling, a Six-pence, a Groat, or 4 *d.* a Three-pence, a Two pence, a Penny.				
Of Copper,	A ½ Penny,	thus writ	0	0	0½
	A Farthing,		0	0	0¼.

Beſides the above-mentioned, we have ſtill in uſe the names of ſome other pieces, which are now but imaginary, *viz.*

		l.	*s.*	*d.*
A Mark,	Value	0	13	4
An Angel,		0	10	0
A Noble,		0	6	8

2461
222/546546
272|4
034|0 remr. 6 3/4/3
11|2

Here 102 is placed perpend.r under 5 instead of 1025 as when 5 brought down

WEIGHTS.

TROY.

24 Grains	make	1 Pennywt.	thus mark'd	dwts.
20 Pennywts.	make	1 Ounce,	thus mark'd	oz.
12 Ounces	make	1 Pound,	thus mark'd	℔.

By *Troy Weight* are weighed Jewels, Gold, Silver, and all Liquors.

APOTHECARIES.

20 Grains	make	1 Scruple,	thus mark'd	℈
3 Scruples	make	1 Dram,	thus mark'd	ʒ
8 Drams	make	1 Ounce,	thus mark'd	℥
12 Ounces	make	1 Pound,	thus mark'd	℔

By theſe Weights, Apothecaries compound their Medicines; but buy and ſell their Drugs by *Avoirdupoiſe.*

AVOIRDUPOISE.

16 Drams	make	1 Ounce,	thus mark'd	oz.
16 Ounces	make	1 Pound,	thus mark'd	℔
28 Pounds	make	1 quarter of a Hund.	thus mark'd	qrs.
4 Quarters	make	1 Hundred,	thus mark'd	C.
20 Hundred	make	1 Ton,	thus mark'd	ton.

This is, at preſent, the common weight of *England*, by which Butter, Cheeſe, and all Groceries, &c. are weighed.

Note. One pound *Avoirdupoiſe*, is equal to 14 *oz.* 11 *dwts.* 15 *grs.* ½ *Troy*; and one ounce *Troy* is equal to 1 *oz.* 1 *dram*, and ſomething above ½ *Avoirdupoiſe.*

WOOL-WEIGHT.

7 Pounds	make	1 Clove,	marked as written.
2 Cloves		1 Stone,	
2 Stones		1 Todd,	
6 Todds and ½		1 Wey,	
2 Weys		1 Sack,	
12 Sacks		1 Laſt,	

LEAD.

C.
19½ make a Fodder.

MEASURES.

WINE.

2 Pints	make	1 Quart,	thus mark'd	*Qrt.*
4 Quarts		1 Gallon,		*Gall.*
63 Gallons		1 Hogſhead,		*Hdd.*
2 Hogſheads		1 Pipe,		*Pipe.*
2 Pipes		1 Ton,		*Ton.*

BEER and ALE.

2 Pints	make	1 Quart,	thus mark'd	*Qrt.*
4 Quarts		1 Gallon,		*Gall.*
9 Gallons		1 Firkin,		*Firk.*
2 Firkins		1 Kilderkin,		*Kild.*
2 Kilderkins		1 Barrel,		*Bar.*
3 Barrels		1 Butt,		*Butt.*

Note, 8 Gallons make 1 Firkin of Ale.

DRY.

3 Pints	make	1 Gallon	thus mark'd	*Gall.*
2 Gallons		1 Peck,		*Peck.*
4 Pecks		1 Buſhel,		*Buſh.*
4 Buſhels		1 Coom,		*Coom.*
2 Cooms		1 Quarter,		*Qr.*
5 Quarters		1 Wey,		*Wey.*
2 Weys		1 Laſt,		*Laſt.*

LONG.

3 Barley-Corns	make	1 Inch,	mark'd as writ.	* 1760 Yards make a Mile.
12 Inches		1 Foot,		
3 Feet		1 Yard *,		
5 Yards and ½		1 Pole or Perch,		
40 Poles		1 Furlong,		
8 Furlongs		1 Mile,		

LAND.

40 Square Perches	make	1 Rood.
4 Roods		1 Acre.

Note, That a Geometrical Pace is 5 Feet, and that there are 1056 ſuch Paces in an *Engliſh Mile.*

CLOTH.

4 Nails	make	1 Quarter,	mark'd	*qr.*
4 Quarters		1 Yard,		*yd.*

Note, An Ell *Flemiſh* is 3 *qrs.* of a Yard: an Ell *Engliſh* is 5.

12	make	1 Dozen.
12 Dozen		1 Small Groſs.
12 Small		1 Great Groſs.

CHAP. II.

Of REDUCTION.

REDUCTION is but an application of Multiplication and Diviſion: For,

Firſt. All great names are brought into ſmall, by multiplying with ſo many of the little ones as make one of the great.

Secondly, All ſmall names are brought into great, by dividing by ſo many of the little ones as make one of the great.

Thirdly, To change one ſort of money, or Weight, *&c.* into another, is only to bring both into one name, and to divide the one by the other.

EXAMPLES of each Sort.

(Firſt Sort.)

l. *s.* *d.*
In 4295 12 3 how many Farthings?
'tis multiplied by 20 *becauſe* 20 s. *make* 1 l. Note, *The* 12 s. *are taken in*
———
85912
'tis multiplied by 12, *becauſe* 12 d. *make* 1 s. *And here the* 3 d. *are added.*
———
1030947
'tis multiplied by 4, *becauſe* 4 *Farthings make* 1 *Penny.*
———
Anſwer, 4123788 *Farthings.*

(Second Sort.)

In 4123788 Farthings, how many Pounds?

4)4123788(

12)1030947(3

2|0)8591|2(

4295 *l.* 12 *s.* 3 *d. facit.*

We divide here by 4, 12, and 20, for the ſame reaſons that we multiplied by them in the firſt example.

(Third Sort.)

Change 45967 *French* Crowns, at 4 *s.* 6 *d.* each into Guineas

s. *d.*
4 6
2

Six-pences in a French*Crown.* } 9

21
2

42 { *Six-pences in a Guinea.*

45967
9

42)413703(9850 *Guineas.*
378
357
336
210
210
... 3 *Six-pence.*

More E X A M P L E S.

In 85647 Guineas, how many Pounds Sterling?
Anſwer, 89929 *l.* 7 *s.*

In 59463 Marks, at 13 *s.* 4 *d.* how many Pounds?
Anſwer, 39642 *l.*

How many Dollars at 4 *s.* 4 *d.* are there in 89573 Piſtoles, at 17 *s.* 6 *d*?
Anſwer, 361737 Dollars, and 6 *d.* remain.

In 82 *lb.* 9 *oz.* 10 *dwts.* 5 *grs.* how many Grains?
Anſwer, 476885 Grains.

In 56 *Tons*, 13 *C.* 2 *qrs.* 21 *lb.* 7 *oz.* 11 *drams*, how many Drams?
Anſwer, 32505211 Drams.

How many minutes ſince the birth of our Saviour, it being 1766 years?
Anſwer, 925056000 Minutes.

How many Gallons are there in 7569 Tuns?
Anſwer, 1907388 Gallons.

Note, Theſe queſtions reverſed, will make as many more good examples.

CHAP. III.

Of PROPORTION, *or the* RULE *of* THREE, *Direct, Indirect, and Double.*

THIS Rule is called the RULE of THREE, becauſe by three numbers given we find a fourth number ſought; which, when the Proportion is direct, muſt always bear the ſame ratio, or proportion, to the third number, as the ſecond bears to the firſt.

The chief difficulty of this Rule lies in ſtating its queſtions. For your direction therefore obſerve, that, of the three given numbers, two always contain a Suppoſition, and the third a Demand.

The Number then on which the Demand lies, muſt always be the third in your ſtating; of the other two you will be ſure to find one of the ſame quality with the ſaid third; which being made your firſt, the number left, muſt, of conſequence, fall in the ſecond place, which ſecond number will alſo always be of the ſame kind with the fourth, or number ſought.

The queſtion being thus ſtated, you muſt (if they are not already ſo) bring your firſt and third numbers into one name, and your ſecond (if of ſeveral denominations) into its loweſt term; then multiplying your ſecond and third numbers together, and dividing the product by your firſt, the quotient will be the anſwer of your queſtion, in the ſame denomination you left your ſecond number. *See the Examples.*

EXAMPLES.

(*Ex.* 1*ſt.*) How many Ounces may I buy for 8 *l.* if I give after the rate of 2 *l.* for 18 Ounces?

l. *oz.* *l.*
If 2 buy——18——what will 8?
8
2)144(

Anſwer, 72 *oz.*

(*Ex.* 2*d.*) What muſt I give for 122 Ells of Holland, if I pay after the rate of 34 *l.* for 138 Ells?

If 138 coſt—34—what will 122?
34
488
366
l. *s.* *d.*
138)4148(30 1 $1\frac{3}{4}$
414
···8
20
138)160(1
138
·22
12
138)264(1
138
126
4
138)504(3
414
·90

The Remainder 8, *being Parts of a Pound, are multiplied by* 20, *to ſee what Shillings they will produce; and ſo the other Remainders by* 12, *and* 4, *to bring out the odd Pence and Farthings.*

This way of valuing the Remainder, is more fully explained in the ſeventh ſort of Reduction of Vulgar Fractions. p. 150

(Ex. 3d.) How many Yards of Muſlin can I buy for 42 *l.* 12 *s.* if $2\frac{1}{2}$ Yards come to 19 *s.* 6 *d?*

s. d. — *Yards* — *l. s.*
If 19 6 buy——$2\frac{1}{2}$——what will 42 12
2 — 2 — 20
39 — 5 — 852
2
1704
5
39)8520(218
78
72
39
330
312
18

2)218(

Anſwer, 109 *Yards.*

PROOF.

Of every four numbers in direct proportion, the product of the two means multiplied into each other, will be equal to the product of the two extremes ſo multiplied; therefore multiplying your firſt by your fourth number found, and comparing it with the produce of your ſecond by your third, if the products agree, the work is right. Thus, in the firſt example, 72 (the fourth number found) multiplied by 2 (the firſt number) gives 144, equal to the produce of 8 times 18, the ſecond and third numbers.

And from hence ariſes the invention of the foregoing rule, for finding the fourth number; for if the ſecond number, multiplied by the third, be equal to the firſt multiplied by the fourth, it is plain that if the product of the ſecond and third be divided by the firſt, the Quotient muſt be the ſaid fourth number; becauſe every Dividend muſt be equal to the produce of its Diviſor and Quotient, the Remainder (if any) being alſo conſidered.

Sums in this rule may alſo be proved by a back-ſtating; thus reverſing the third Example.

	l.	*s.*		*yards.*		*s.*	*d.*		*yards.*
If	42	12	will buy	109,	then	19	6	will buy	$2\frac{1}{2}$
	20			39		2			
	852			981		39			
	2			327					
	1704								

1704)4251($2\frac{1}{2}$ *yards.*
3408
·843
2
1686
18 } *Remainder of the other ſtating added.*
1704)1704(1
···0

Though, in the former of the foregoing proofs, the manner of working the *Rule of Three* is ſufficiently accounted for, and the reaſon of the operation plainly enough laid down; yet, as this may juſtly be looked upon as the main Rule of Arithmetic, and as the ſame application of proportion runs through almoſt all its other branches, it may, perhaps, help to make it ſtill more evident, to look back again upon the firſt example, and obſerve it in another light. The queſtion there ſtated was, If 2 *l.*—18 *oz.*—8 *l.* Now had it been, If 1 *l.*—18 *oz.*—8 *l.* 'tis clear, that if 1 *l.* would buy 18 *oz.* then 8 *l.* would buy 8 times as many, that is, 144 *oz.* But then, as this is upon the ſuppoſition that 1 *l.* is the price of 18 *oz.* ſo if the price of the 18 *oz.* is doubled, that is, made 2 *l.* 'tis plain 144 *oz.* will then be twice the Quantity that 8 *l.* will buy, and muſt therefore be divided by 2, to give the true Anſwer. Again, had the ſuppoſed Price of 18 *oz.* been 4 *l.* then would 8 *l.* have bought but a fourth part of 144 *oz.* *viz.* 36 *oz.* conſequently, in what proportion ſoever the firſt number increaſes, in the ſame proportion muſt the fourth number decreaſe; which is plainly effected by dividing by the firſt number.

Note, In the operation of the *Rule of Three,* the firſt and third numbers, after preparation, are no more regarded, as of any denomination; but are multiplied and divided with, only as abſolute numbers, increaſing and decreaſing the middle number in juſt proportion.

Note alſo, That what has been ſaid of the reaſon of working the *Rule of Three,* ought alſo to be conſidered in the operations of *Intereſt, Rebate, Fellowſhip, Exchange,* and all other applications of proportion.

We now proceed to ſome other practical obſervations.

1. If you would know at what rate you muſt ſell out your goods by retail, ſo as to make a propoſed gain by the whole, add the money you would gain to the ſum the whole goods coſt you; and then ſtate your queſtion thus: *If the whole be ſold for the total of the Coſt and Gain, what muſt any part?*

EXAMPLE.

Suppoſe I would, by the ſelling of 32 yards of Broad Cloth, which coſt me 40 *l.* gain 5 *l.* for what muſt I ſell it *per* yard? Then,

```
      yards.          l.          yard.
   If 32 be ſold for 45, what will 1 ?
l.                    1
40 Coſt.              — l.  s.  d.
 5 Gain.          32)45(1   8   1½ facit.
—                    32
45 Total.            —
                     13
                      20
                     ——
                  32)260(8
                     256
                     ———
                     ..4
                      12
                      —
                  32)48(1
                     32
                     —
                     16
                      4
                     —
                  32)64(2
                     64
                     —
                     .0
```

(2*d.*) Or if damage having happened to the Cloth, 5 *l.* were to have been loſt by the whole, then the ſaid 5 *l.* muſt have been ſubtracted from the coſt, and the remainder made the ſecond number, as before.

(3*d.*) If you would barter or exchange your goods for others, firſt find the value of your own, and then ſee what quantity of the others the ſum will purchaſe.

EXAM-

EXAMPLES.

What quantity of Pepper, at 3 *s.* 6 *d.* per ℔. may I have in exchange for 426 C. of Tobacco, at 53 *s.* *per* C?

C. *s.* C.
Firſt, If 1 —— 53 —— 426
53
1278
2130
Anſwer, 22578 *Shillings*.

s. *d.* ℔. *s.*
Then, If 3 6 —— 1 —— 22578
2 2
7 7)45156(6
Anſwer, 6450 *Pounds* $\frac{6}{7}$

ABBREVIATION.

The work of ſome ſtatings may be much ſhortened, by dividing their firſt and ſecond, or firſt and third numbers, by any figure, ſo that nothing may remain.

EXAM-

EXAMPLE.

l. *Men.* 6) *l.*

6| If 192|00 *would pay* 5|00 how many would 17664 pay ?

4| 32 4| 2944

8 736

5

8)3680(

Anſwer, 460 *Men.*

The numbers of the above ſtating are abbreviated, by firſt cutting off the Cyphers from the firſt and ſecond numbers, and then dividing the firſt and third numbers by 6 and 4.

The reaſons of this abbreviation may be ſeen in the fifth ſort of Reduction of Vulgar Fractions. p. 147

More Queſtions in the DIRECT RULE *of* THREE.

The cloathing of a Regiment of 740 men, comes to 3000 *l.* how much is that for each man ?

Anſwer, 4 *l.* 1 *s.* 0 *d.* ¾

How long ſhall I be laying up 10000 *l.* if I put by ½ a Guinea a week ?

Anſwer, 366 *years,* 15 *weeks,* 4 *days.*

500 Seamen are to have 4 *d.* ½ *per* day each, what will pay them for 23 months ?

Anſwer; 6037 *l.* 10 *s.*

What will an Eſtate of 4000 *l.* *per annum,* allow a Gentleman to ſpend a day ?

Anſwer, 10 *l.* 19 *s.* 2 *d.*

A Gentleman's daily expences are 13 *s.* 7 *d.* above which, he yearly lays up 600 Nobles, at 6 *s.* 8 *d.*

each, what is his Eſtate worth *per annum*?

Anſwer, 447 *l.* 17 *s.* 11 *d.*

A man owing 736 *l.* 10 *s.* compounds with his Creditors for 7 *s.* 9 *d.* *per l.* what will that amount to?

Anſwer, 285 *l.* 7 *s.* 10 *d.* $\frac{1}{2}$

If, at 5 *s.* *per* ell, I gain 8 *l.* *per Cent.* by my Cloth, what ſhall I gain *per Cent.* if I ſell the ell at 6 *s.* 3 *d.*?

Anſwer, 35 *l.*

If Tobacco, which coſt 13 *d.* *per* ℔. be ſold for 18 *d.* *per* ℔. what is gained *per Cent*?

Anſwer, 38 *l.* 9 *s.* 2 *d.* $\frac{3}{4}$.

Bought 5 pieces of Holland, each containing 56 ells *Flemiſh*, at 3 *s.* 2 *d.* *per* ell, what ſhall I gain in the whole, if I ſell it for 5 *s*, 8 *d.* *per* ell *Engliſh*?

Anſwer, 3 *l.* 5 *s.* 4 *d.*

I have by me 96 ℔. of Cloves, which coſt me 58 *l.* but ſome damage having happened to them, I am willing to loſe 8 *l.* in the whole; at what rate muſt I ſell them *per* ounce?

Anſwer, 7 *d.* $\frac{3}{4}$.

A Merchant ſends over to *France* 482 tons of Lead, at 4 *l.* 10 *s.* *per* Fodder, *i. e.* 19$\frac{1}{2}$ *C.* what quantity of Wine, at 30 *l.* *per* Pipe, may he expect in return?

Anſwer, 74 Pipes, 19 Gallons.

A Merchant ſends to *Spain* 1300 pieces of Broad Cloth, each piece 47 yards, at 15 *s.* 6 *d.* *per* yard, to have returns from thence; the one half in Wine, at 65 *l.* *per* ton; and the other half in Oranges, at 3 *l.* 10 *s.* *per* Cheſt; what quantity of each will he have?

Anſwer, { 364 Tons, 1 Hogſhead of Wine.
6764 Cheſts of Oranges.

Two men part, at the ſame time, from the ſame place; the one travels north 32 miles a day, the other 36 miles a day ſouth; how long will it be before they are 2000 miles aſunder, ſuppoſing them to travel 12 hours each day?

Anſwer, 29 days, 4 hours, 56$\frac{1}{2}$ minutes.

The Indirect RULE *of* THREE.

IN the *Indirect Rule of Three,* the numbers are in reciprocal proportion, that is, the fourth number to be found, is to bear the ſame ratio to the ſecond as the third does to the firſt, but in an inverted order; that is, the greater the third term is in reſpect to the firſt, the leſs muſt the fourth be in reſpect to the ſecond.

This rule differs, in its operation, from the Direct, in that, after the queſtion is ſtated, and the numbers of the ſtatings prepared (as in the Direct Rule) your firſt and ſecond muſt be multiplied together, and your third number be your Diviſor. The Quotient, as before, will be the anſwer.

EXAMPLES.

(Ex. 1ſt.) What number of men muſt be employed, to finiſh, in 12 days, what 43 men would be 35 days about?

```
   days.      men.      days.
If 35 ———— 43 ———— 12
   43
   ——
   105
  140
  ————
12)1505
  ————
```

Anſwer, 125 *Men.*

(Ex. 2d.) How many yards of Stuff 3 *qrs.* wide, will hang a Room which requires 420 yards of 5 *qrs.* wide?

qrs. *yards.* *qrs.*
If 5 —— 420 —— 3
5
3)2100(
Anſwer 700

The reaſon of this operation will appear plain (after what has been ſaid in the Direct Rule) by conſidering the laſt example. Now it is clear, that if the Stuff, being 5 quarters wide, there are 420 yards required, then were the Stuff but 1 quarter wide, 5 times 420 yards, *viz.* 2100 yards muſt be allowed; conſequently, if the Stuff be 3 quarters wide, one third part of thoſe yards will be ſufficient; therefore 2100 divided by 3, will give the true anſwer required, *viz.* 700 yards.

To know whether a Queſtion belongs to the Direct *or* Indirect Rule *of* Three.

Obſerve, If the third number, being more than the firſt number, requires more, or, being leſs, requires leſs, it is *Direct*; but if the third number, being more, requires leſs, or being leſs, requires more, it is *Indirect*.

Or, without any regard to the diſtinction of *Direct* and *Indirect*; if *more* is required, let the leſſer of the two extremes be the Diviſor, if *leſs*, the greater.

More Queſtions in the Indirect Rule of Three.

If I lend *A* 136 *l.* for 3 months, how long muſt I keep 42 *l.* of his, to requite myſelf?

Anſwer, 9 months, 2 weeks, 6 days.

If 46 Clerks in 32 days finiſh a piece of writing, in what time would 55 Clerks accompliſh the ſame?

Anſwer, 26 days, 9 hours, 9 minutes.

A Garriſon, conſiſting of 1539 men, being beſieged, have proviſions only for 12 days; but it being neceſſary they ſhould hold out 3 weeks, how many men muſt be ſent out?

Anſwer, 660 men.

1539
12
2/18468/ 1539
879 21
660

The DOUBLE RULE *of* THREE.

Queſtions in this Rule have five numbers propoſed, and are frequently anſwered by two ſtatings, tho' they may be performed by one, as ſhall be ſhewn hereafter.

EXAM-

EXAMPLES.

(*Ex.* 1*ſt.*) The carriage of 32 hundred weight (56 miles) comes to 12 *s.* After the ſame rate, what muſt I pay to have 78 hundred weight carried (94 miles?)

C. — *s.* — *C.*
Firſt, *If* 32 —— 12 —— 78
12
s. *d.*
32)936(29 3 *facit.*
64
296
288
8
12
32)96(3
96
0

Miles — *s.* *d.* — *Miles.*
Then, *If* 56 —— 29 3 —— 94
12
351
94
1404
3159
56)32994(589
280
499
448
514
504
10

12)589(1
2|0|4|9(

Anſwer, 2 *l.* 9 *s.* 1 *d.*

Note, The ſolution had been the ſame, if the miles had been made firſt and third numbers of the firſt ſtating; and the *C.* weights the firſt and third numbers of the laſt.

Note alſo, this example may be done by one ſtating, thus:

```
      C.          s.         C.
  If  32 ——————— 12 ——————— 78
      56 Miles.              94 Miles.
     ————                   ————
     192                    312
    160                    702
     ————                   ————
    1792                   7332
                             12
                         ————————  s.  d.
                  1792)87984(49 . 1
                       7168       or
                       ————————  l. s. d.
                       16304    2.9.1
                       16128
                       ————
                        .. 176
                           12
                         ————
                  1792)2112(1
                       1792
                       ————
                       . 320
```

(*Ex. 2d.*) How many men muſt be employ'd to reap 420 Acres in 17 days, if there were required 37 men to reap 54 acres in 5 days ?

```
                         Acres.      Men.      Acres.
              Firſt, If 54 ————— 37 ————— 420
                                               37
                                             ————
                                             2940
                                             1260
       Days.        Men.        Days.        ————
Then, If 5 ————— 287 ————— 17          54)15540(287
                   5                        108
                 ————                      ————
              17)1435(84                    474
                 136                        432
                 ————                      ————
                   75                       420
                   68                       378
                 ————                      ————
Anſw. 84 Men.       7                        42
```

Note, If you would work ſuch queſtions of the *Double Rule of Three,* as have one of their proportions indirect, by one ſtating, you muſt multiply the third number of your ſtating, by that number you would otherwiſe have placed under your firſt ; and your firſt number by that you would have placed under the third ; as in the following Example.

EXAMPLE.

Acres. *Men.* *Acres.*

If 54 ——— 37 ——— 420 } *The number of days*
* 17 5 } *which have relation to*
— — } *the* 54 *Acres.*

378 2100
54 37
—— ——
918 14700
6300
——— *Men.*
918)77700(84 *anſwer as before.*
7344 .
———
. 4260
3672
———
. 588

* *The number of days which have relation to the* 420 *Acres.*

Dr. *Harris*, in his *Lexicon Technicum*, teaches another way of ranging the five Numbers given in a *Double Rule of Three Queſtion*; which, according to the following Rules, gives the ſolution, whether the proportion be direct or indirect. Take his method in his own words:

I. "Obſerving, that the given terms are always "five, whereof three are conditional and antecedent, "or ſuppoſititious, the other two demand the queſtion, "and are conſequents anſwering ſome of the former "antecedents; inſomuch, that, with the anſwer, "there will be as many conſequents as antecedents, "which muſt match one another in the ſame denomi-"nation exactly.

II. "For the right placing of the queſtion and "terms, the three terms of the conditional part are "duly to be regarded: Let that which is the princi-"pal cauſe of Loſs or Gain, Increaſe or Decreaſe, Ac-"tion or Paſſion, be put in the firſt Place; and that

“ which betokeneth the ſpace of time, diſtance of place, *&c.* be put in the ſecond place; and the remaining part in the third. The conditional part thus ſtated, the other two terms, wherein the demand lies, muſt be placed ſo under the former terms, that they may correſpond one with another.

RULE 1.

“ Then, if the blank, or place ſought, fall under the third term, multiply the three laſt terms for a Dividend, and the two firſt for a Diviſor; and the Quotient gives the ſixth term required.

RULE 2.

“ But if the blank fall under the firſt or ſecond term, multiply the 1ſt, 2d, and 5th terms for a Dividend, and the 3d and 4th for a Diviſor: The Quotient gives the Anſwer.

EXAMPLE 1.

“ If 12 Rods of ditching be done by 2 men in 6 days, how many Rods ſhall be wrought by 8 men in 24 days?

Anſwer, 192.

The numbers being ſtated as before directed, they will ſtand thus, the blank falling under the third place.

Men.	*Days.*	*Rods.*
2———	6———	12
8———	24	

Therefore, by the firſt Rule, the three laſt terms, *viz.* 12, 8, and 24, being multiply'd into each other, give 2304 for a Dividend; and the two firſt terms, *viz.* 2 and 6, multiply'd together, give 12 for a Diviſor, which quotes 192, the anſwer.

EXAM-

EXAMPLE 2.

If 2 men work 12 Rods in 6 days, how many men will work 192 Rods in 24 days?

Men.	*Days.*	*Rods.*
2	6	12
	24	192

Here, according to the ſecond Rule, the firſt, ſecond and fifth terms multiply'd, give 2304, which divided by 288, the product of the third and fourth numbers, quotes 8 the anſwer.

More EXAMPLES.

If 400 pecks of Corn will ſerve 32 horſes 108 days, what quantity will 500 horſes eat in 20 days?

Anſwer, $1156\frac{2}{5}$ *pecks.*

If 56 gallons of Drink ſerve 25 perſons 120 days, how long will 200 gallons ſerve 12 perſons?

Anſwer, $892\frac{6}{7}$ *days.*

If a certain quantity of proviſion ſerves 30 men for 50 days with 24 ounces each, how much muſt 90 men have apiece when they are obliged to make the ſame proviſion ſerve them 60 days?

	Ounces.	*Drams.*
Anſwer,	6	$10\frac{2}{3}$.

Mr. *Gardiner* has publiſhed, in the explication of his Tables of Logarithms, a peculiar method of reſolving the ſeveral kinds of this Rule, (taken from the Papers of *W. Jones*, Eſq; F. R. S.) which is not only excellent in the uſe of Logarithms, but alſo in the way of common Arithmetic.

I. "Set down the terms expreſſing the condition of " the queſtion in one line, and in any order.

2. " Under each conditional term ſet its corre-
" ſponding one, in another line.

3. " Multiply the producing terms of one line, and
" the produced term of the other line, continually,
" and the reſult for a dividend.

4. " Multiply the remaining terms continually, and
" let their product be the diviſor.

5. " The quotient of this diviſion will be the term
" required.

" By producing terms here is meant, whatſoever ne-
" ceſſarily and jointly produce any effect: As the
" cauſe, and the time; length, breadth and depth;
" buyer and his money; ſeller and his goods; all ne-
" ceſſarily inſeparable in producing their ſeveral ef-
" fects."

In a queſtion where a term is underſtood, and not expreſſed, (as in the third Example) that term may be repreſented by Unity.

This Rule is ſo general, as to comprehend all caſes that come under the common Rule of Three, whether direct, or inverſe; whether ſingle, or any how compounded; ſo that queſtions in any of the following Rules, where proportion is uſed, may be readily ſolved thereby.

EXAMPLES.

Ex. 1. Take the ſame as in page 38. Putting down the terms expreſſing the condition in the firſt line, and under each term its correſponding one in the ſecond, filling the blank ſpace (or term ſought) with a capital Q thus.

	Men.		*Days.*		*Acres.*
If	37	require	5	to reap	54
	Q		17		420

Here 'tis plain, that the produced terms are the acres, and the men and days are the terms producing the quantity of acres reaped; therefore looking in page 128 for the characters here uſed, and particularly the latter part of that for diviſion, it will be that

$$Q = \frac{420 + 37 + 5}{54 + 17} = \frac{70 + 3 + 5}{9 + 7} = \frac{12950}{153} = 84 \text{ Men.}$$

Ex. 2. Take that in page 41, thus:

	Rods.	*Men.*	*Days.*
If	12 ——	2 ——	6
	Q	8	24

Here the produced terms are the rods, and the producing terms the men and days, as before.

$$Q = \frac{12 + 8 + 24}{2 + 6} = 8 + 24 = 192 \text{ Rods.}$$

Note, When any of the terms in the dividend are the ſame as in the diviſor, they may be ſtruck out in both; or when any in both may be equally divided, the quotients may be put inſtead of thoſe terms, as in the firſt example.

Ex. 3. If 54 men can build a Fort in 18 days, when working 17 hours in each day; in how many days will 120 men build the ſame, in working but 12 hours each day?

	Men.	*Days.*	*Hours.*	*Fort.*
If	54 ——	18 ——	17 ——	1
	120	Q	12	1

Here the Fort being 1 is the term produced, and the men, days and hours, the producing terms; therefore

$$\begin{array}{r} \textit{Ds. Hrs. Min.} \\ Q = \frac{54 + 18 + 17}{120 + 12} = \frac{27 + 17}{40} = \frac{459}{40} = 11 : 5 : 42 \end{array}$$

Ex. 4. What is the Intereſt of 572 *l.* for 8 months, at 5 *l. per Cent per Annum?* See page 68.

If 100 *l.*———5 *l.*———12 Months.
572 Q 8

Here the Intereſt is the term produced, and the principal and months the producing terms; therefore

$$Q = \frac{572+8+5}{100+12} = \frac{572}{30} = 19\tfrac{2}{30} = 19\,l.\ 1\,s.\ 4\,d.$$

Ex. 5. If 250 men in 5 days, working 16 hours each day, will dig a trench 240 yards long, 5 yards wide, and 4 deep; in how many days, working but 10 hours in each day, will 24 men dig a trench 360 yards long, 4 wide, and 3 deep?

	Men.	*Days.*	*Hours.*	*long.*	*wide.*	*deep.*
If	250———	5———	16———	240———	5———	4
	24	Q	10	360	4	3

Here the produced terms are the trenches, conſiſting of length, width and depth, and the men, days and hours, the producing terms; therefore

$$Q = \frac{360+4+3+250+5+16}{240+5+4+24+10} = 75 \text{ days}.$$

As the Tables of Logarithms, when truly printed, are of excellent uſe to the proficients in Arithmetic; and having above an occaſion given for mentioning thoſe publiſhed by Mr. *Gardiner*, I think my readers ſhould be informed of their being the moſt correct, as well as complete, of any extant; and that he has publicly advertiſed ten ſhillings reward to be ſhewn one error, even of a logarithm not within the half unit of the loweſt place, or of a difference untrue between them, tho' given to every ten ſeconds of the Quadrant.

His Explication contains many curious ſolutions in logarithms, and very eaſy examples of getting the logarithm of any number, and the number to any logarithm, true to 19 places of figures, by tables therein given to 20 places; beſides thoſe of all numbers from 1 to 102100, with parts for two figures farther, every where in the ſame openings as the logarithms of the firſt five or ſix figures.

CHAP.

CHAP. IV.

Of PRACTICE, *and* TRETT *and* TARE.

BY PRACTICE Merchants compendiouſly caſt up the Price of their Commodities.

And by TRETT and TARE, deduct their Allowances.

Both theſe Rules will be beſt explained by Examples; and both require the perfect Knowledge of the following

TABLE.

The even Parts

Of a Pound.		
s.	*d.*	
1	0	$\frac{1}{20}$
1	8	$\frac{1}{12}$
2	0	$\frac{1}{10}$
2	6	$\frac{1}{8}$
3	4	$\frac{1}{6}$
4	0	$\frac{1}{5}$
5	0	$\frac{1}{4}$
6	8	$\frac{1}{3}$
10	0	$\frac{1}{2}$

The even Parts

Of a Shilling.	
d.	
1	$\frac{1}{12}$
1$\frac{1}{2}$	$\frac{1}{8}$
2	$\frac{1}{6}$
3	$\frac{1}{4}$
4	$\frac{1}{3}$
6	$\frac{1}{2}$
Of a Pen.	
Far.	
1	$\frac{1}{4}$
2	$\frac{1}{2}$

The even Parts

Of a Ton.	
C.	
2	$\frac{1}{10}$
2$\frac{1}{2}$	$\frac{1}{8}$
4	$\frac{1}{5}$
5	$\frac{1}{4}$
10	$\frac{1}{2}$
Of a Hund.	
℔.	
14	$\frac{1}{8}$
16	$\frac{1}{7}$
Of $\frac{1}{2}$ *C.*	
℔.	
7	$\frac{1}{8}$
8	$\frac{1}{7}$
14	$\frac{1}{4}$

Before

Before we proceed to the rules of Practice, it will be proper to ſhew the manner of multiplying and dividing numbers of ſeveral denominations, by a ſingle figure, without Reduction.

Example of MULTIPLICATION.

	l.	*s.*	*d.*	
Multiply	5 .	8 .	4	*by* 8.
			8	
Product	43 .	6 .	8	

Here firſt, the 4*d.* multiplied by 8, gives 32 *d.* equal to 2 *s.* 8 *d.* therefore ſetting down 8 in the place of pence, we carry 2 to the produce of the ſhillings. Again, 8 times 8 *s.* being 64 *s.* and the 2 carried making it 66 *s.* or 3 *l.* 6 *s.* the 6 is ſet down in the place of ſhillings, and the 3 carried on to the Produce of the Pounds. Laſtly, 8 times 5 *l.* being 40 *l.* and the 3 carry'd making 43 *l.* the whole product appears to be 43 *l.* 6 *s.* 8 *d.*

Example of DIVISION.

	l.	*s.*	*d.*	
Divide	43 .	6 .	8	*by* 8.
8)	43 .	6 .	8 (	
Quotient	5 .	8 .	4	

In dividing the above ſum, we find, firſt, that 8 will go 5 times in 43 *l.* and that 3 will remain; therefore ſetting down 5 in the place of pounds, we multiply the ſaid remainder by 20, and taking in the 6 odd ſhillings, the produce is 66 *s.* in which again, the diviſor going 8 times, and 2 remaining, we ſet down 8 in the place of ſhillings, and multiply the 2 remaining by 12, which, with the 8 odd pence taken in, produces 32 *d.* in which, laſtly, the Diviſor 8 going exactly 4 times, 4 is ſet in the place of pence, and the whole quotient will be found to be 5 *l.* 8 *s.* 4*d.*

Rules of PRACTICE, *with Examples.*

1. When the Price is ſuch a number of pence as make an even part of a ſhilling, divide the quantity of goods by ſuch part, and the quotient will be the anſwer in ſhillings.

EXAMPLE.

Suppoſe you would know the price of 5296 ounces of Gum, at 6 *d. per* ounce; then

6 d. being $\frac{1}{2}$ *of a Shilling* } 5296
——
2648 *ſhillings is the Anſwer.*

For were the price 12 *d.* or 1 *s.* the number of ounces would be the number of ſhillings they would coſt: Therefore the price being 6 *d.* the $\frac{1}{2}$ of a ſhilling, the anſwer muſt be but $\frac{1}{2}$ ſo many ſhillings. So at 1 *d.* 1$\frac{1}{2}$ *d.* 2 *d.* 3 *d.* or 4 *d.* 'tis but the $\frac{1}{12}$, $\frac{1}{8}$, $\frac{1}{6}$, $\frac{1}{4}$, or $\frac{1}{3}$ parts of the quantity of the goods, and the quotient (as aforeſaid) will be the anſwer in ſhillings; which divided by 20, gives the pounds.

Note, If any thing remains in dividing by the ſaid parts (pence being the next denomination) it muſt be multiply'd by 12, the pence in 1 ſhilling; and the product divided again by the ſaid part; and if any thing remains there, it muſt be multiply'd by 4, and ſtill divided by the ſaid part, to produce farthings.

EXAMPLE.

oz. *d.*
5243 at $1\frac{1}{2}$ *per* Ounce.
d.
$1\frac{1}{2}$ $\frac{1}{8}$ | 65|5 . $4\frac{1}{2}$

Anſwer. - - 32*l.* 15*s.* $4\frac{1}{2}$

2. If the pence of the price are not an even part of a ſhilling, it will require more than one diviſion, to find the coſt of the goods. Thus at 5 *d.* or any other number of pence between 6 and 12.

EXAMPLES.

℔. *d.* ℔. *d.*
527 at 5 *per* lb. 436 at 9 *per* lb.
d.
$4\frac{1}{3}$ | 175 8 $6\frac{1}{2}$ | 218
$1\frac{1}{4}$ | 43 11 $3\frac{1}{2}$ | 109

21|9 7 32|7
Anſw. 10 *l.* 19*s.* 7*d.* *Anſw.* 16*l.* 7*s.*

In the firſt of theſe, 5*d.* is taken, thus: 4*d.* is the $\frac{1}{3}$ of a ſhilling, or the top-line, and 1*d.* the $\frac{1}{4}$ of 4 pence.

In the ſecond, the $\frac{1}{2}$ of a ſhilling is firſt taken for 6*d.* and then the $\frac{1}{2}$ of that 6*d.* for 3*d.*

3. If the price is both pence and farthings; for the pence, as before; and for the farthings, take the even parts of a penny: thus,

EXAM-

EXAMPLE.

		oz.	*d.*
		532	at 1¼ *per* Ounce.
1 *Penny,*	$\frac{1}{12}$	44 .	4
1 *Farthing,*	¼	11 .	1
		5|5 .	5
Anſwer,		2*l.* 15*s.*	5*d.*

4. When the price is more than 1 *s.* and leſs than 2 *s.* leave the top line for the ſhilling, and take your parts as before, for the remaining pence.

EXAMPLE.

d.		lb.	*d.*
		5437	at 19½ *per* lb.
6	½	2718 .	6
1½	¼	679 .	7½
		883|5 .	1½
	Anſwer,	441 . 15*s.*	1½

5. If the price is two or more ſhillings, with pence, &c. either take the even parts of a pound from your top line for the ſhillings, which will give you the price in pounds; or multiply your top line by the number of ſhillings, which will give the anſwer in ſhillings, ſtill working for the pence as before. See both ways in the example.

EXAMPLE.

```
               oz.      s. d.        oz.      s. d.
              5263 at 2 . 6 per Oz.  5263 at 2 . 6 per Oz.
 s. d.        ————                      2
 2 6 | ⅛ |    657 . 17 . 6           ————————
                         d. |   |    10526
                         6  | ½ |     2631  . 6
                                     ——————————
                                     1315|7 . 6
                                     ——————————
                       Anſwer, 657l.   17s. 6d.
```

By the firſt way, it is thus done: Had the price been 1*l.* or 20*s.* it is plain it would have come to as many pounds as there were ounces; therefore at 2*s.* 6 *d.* which is the ⅛ of a pound, it muſt come to the ⅛ of ſo much money.

The other is done by ſuppoſing the top line at 1*s.* then for 2*s.* it muſt be twice ſo much; and for the 6*d.* half the top line, or the value of a ſhilling, muſt be added. See both ways in another

EXAMPLE.

```
             ℔.    s. d.               ℔.    s. d.
           5276 at 19 9 per ℔.       5276 at 19 9 per ℔.
 s.        ————                        19
 10 | ½ |  2638                      ——————
 5  | ½ |  1319                      47484
 4  | ⅕ |  1055 . 4             d.   5276
 6d | ⅛ |   131 . 18            6½   2638
 3  | ½ |    65 . 19            3½   1319
           ——————————                ——————
 Anſwer,   5210l.   1s.              10420|1
                                     ———————
                            Anſwer,  5210l. . 1s.
```

6. When the price is any thing between one and two pounds, leave the top line for the pound, and take your parts for the remainder.

EXAMPLE.

s. *d.* 5384 at 22 . 6 *per* ℔.
2 6$\frac{1}{8}$ 673

Anſwer, 6057*l.*

7. When the price is two or more pounds, multiply the top line by the number of pounds; and for the remainder take parts, as before.

EXAMPLE.

℔. *l.* *s.* *d.*
2364 at 4 . 18 . 10$\frac{1}{2}$ *per* ℔,
4

s.		9456		
10	$\frac{1}{2}$	1182		
5	$\frac{1}{2}$	591		
2	$\frac{1}{5}$	236	8	
1	$\frac{1}{2}$	118	4	
6*d*	$\frac{1}{2}$	59	2	
3	$\frac{1}{2}$	29	11	
1$\frac{1}{2}$	$\frac{1}{2}$	14	15	6

Anſwer, 11687*l.* 00*s.* 6*d.*

8 When with the given quantity there are odd parts, ſuch parts muſt be taken from the given price; as in the following examples.

EXAM-

EXAMPLES.

C. *qrs.* *l.* *s.*
365 . 2 at 5 . 5 *per C.*
5

s.		1825		
5	$\frac{1}{4}$	91	5	
2 *qrs.*	$\frac{1}{2}$	2	12	6
Anſwer,		1918*l.*	17*s.*	6*d.*

C. *qrs.* ℔. *l.* *s.* *d.*
7365 . 3 . 14 at 8 19 6 *per C.*
8

s.		58920		
10 . .	$\frac{1}{2}$	3682	10	
5 . .	$\frac{1}{2}$	1841	5	
4 . .	$\frac{1}{5}$	1473	0	
6*d.* .	$\frac{1}{8}$	184	2	6
2 *qrs.*	$\frac{1}{2}$	. . 4	9	9
1 . .	$\frac{1}{2}$	. . 2	4	10$\frac{1}{2}$
14 ℔.	$\frac{1}{2}$	. . 1	2	5$\frac{1}{4}$
Anſw.		66108*l.*	14*s.*	6*d.*$\frac{3}{4}$

As in the firſt of theſe, for the 2 *qrs.* $\frac{1}{2}$ the price of the *C.* is taken; ſo in the ſecond, not only $\frac{1}{2}$ the price of the *C.* is taken for 2 *qrs.* but alſo $\frac{1}{2}$ the price of 2 *qrs.* for 1 *qr.* and $\frac{1}{2}$ that again for 14 ℔. *See more Examples.*

EXAM-

EXAMPLES.

Tons. *C.* *qrs.* ℔. *l.* *s.* *d.*
573 . 13 . 2 . 18 at 9 . 12 . 10 *per* Ton.
9

		5157		
10*s*	$\frac{1}{2}$	286	10	
2	$\frac{1}{5}$	57	06	
10*d.*	$\frac{1}{12}$	23	17	6
10 *C.*	$\frac{1}{2}$	4	16	5
2	$\frac{1}{5}$		19	3¼
1	$\frac{1}{2}$		09	7½
2 *qrs.*	$\frac{1}{2}$		04	9¾
14 ℔.	$\frac{1}{4}$		01	2¼
2	$\frac{1}{7}$			2
2	$\frac{1}{7}$			2

Anſwer, 5531*l.* 5*s.* 1*d.*¾

℔. *oz.* *dr.* *l.* *s.* *d.*
527 . 13 . 15 at 2 . 8 . 7½ *per* ℔.
2

		1054		
5*s.*	$\frac{1}{4}$	131	15	
2	$\frac{1}{10}$	52	14	
1	$\frac{1}{2}$	26	07	
6*d.*	$\frac{1}{2}$	13	03	06
1½	$\frac{1}{4}$	3	05	10½
8 *oz.*	$\frac{1}{2}$	1	04	03¾
4	$\frac{1}{2}$		12	01¾
1	$\frac{1}{4}$		03	00¼
8 *drs.*	$\frac{1}{2}$		01	06
4	$\frac{1}{2}$			09
2	$\frac{1}{2}$			04½
1	$\frac{1}{2}$			02¼

Anſwer, 1283*l.* 7*s.* 8*a.*

The

The laſt Sum another way.

℔. *oz.* *drachms* *l.* *s.* *d.*
527 13 15 at 2 8 7½ *per* ℔.
48

4216
2108
6 *d.* . ½ . . . 263 . 6
1½ . ¾ . . . 65 . 10½
8 *oz.* . ½ . . . 24 . 3¾
4 . . ½ . . . 12 . 1¼
1 . . ¼ . . . 3 . 0¼
8 *dr.* ½ . . . 1 . 6
4 . . ½ 9
2 . . ½ 4½
1 . . ½ 2¼

2566|7 . 8
Anſwer, 1-83*l.* 7*s.* 8*d.*

Another EXAMPLE.

℔. *Troy.* *oz.* *dwts.* *grs.* *l.* *s.* *d.*
736 . 11 . 12 . 13 at 23 16 4 *per* ℔.
23

2208
1472
10 *s.* . . ½ . . . 368
5 . . . ½ . . . 184
1 . . . ⅕ . . . . . 36 . 16 .
4 *d.* . . ⅓ . . . . . 12 . 5 . 4
6 *oz.* . . ½ . . . . . 11 . 18 . 2
4 . . . ⅓ . . . . . 7 . 18 . 9¼
1 . . . ¼ . . . . . 1 . 19 . 8¼
10 *dwts.* ½ . . . . 19 . 10
2 . . . ⅕ . . . . 3 . 11½
12 *grs.* . ¼ 11¼
1 . . . $\frac{1}{12}$ . . . . ¾

Anſwer, 1755:*l.* 2*s.* 9½*d.*

Note. The foregoing examples, and all other queſtions in *Practice*, may be performed by the *Rule of Three*; *Practice* being only a compendious way of calculating ſuch propoſitions in proportion, as have an unit for their firſt number: But to ſhew the brevity of this way of work, as well as to prove the truth thereof; ſee the laſt example done by the *Rule of Three*, as follows:

By the *Rule of Three* thus:

```
 ℔.      l.   s.  d.            ℔.    oz.  dwt  grs.
If 1 — coſt 23 . 16 . 4 what will 736 . 11 . 12 . 13 ?
 12      20                    12
 ——      ——                   ———
 12      476—                 8843
  20      12                    20
 ——      ———                 ————————
 240     5716                 176872—
  24                              24
 ———                          ————————
  960                         707491
 480                         353715
 ———                          ————————
 5760                         4244941
                                 5716
                              ————————
                             25469646
                             4244941
                           29714587
                          21224705       12
               576|0)2426408275|6)4212 : 14(10
                     1224         2|0)35104 2
                     ..720  Anſwer, 17552. 2. 10¾.
                        1448
                         2962
                         ..827     Three Farthings
                            2515   more than by
                            2116   Practice loſt there
                               4   by the Fractions
                                   remaining.
                    576|0)846|4(1
                          270          PROOF.
```

PROOF.

Sums in this rule may be proved either by taking the parts two ways, as the laſt example but one; or by the *Rule of Three*, as the laſt.

ABBREVIATIONS.

As this whole rule is indeed nothing elſe, it may be thought ſtrange to mention ſhort ways here, in a particular ſection; but as there are degrees of compariſon, ſo even ſhort ways may be more ſhortened.

1. The value of any quantity (the given price of one of which is an even number of ſhillings) may be moſt compendiouſly found, by multiplying the ſaid quantity by $\frac{1}{2}$ their number, doubling the Units of the firſt product for ſhillings, the reſt being pounds.

EXAMPLE.

5276 *Ells of Holland*, at 8 *s. per Ell.*
4
Product, 2110 : 8

2. If the price is an odd number of ſhillings, it is but doing as before, and adding $\frac{1}{20}$ of the top line to the Product.

EXAMPLE.

Ells. *s.*
5278 at 15 *per Yard.*
7
3694 : 12
1 *s.* $\frac{1}{20}$ 263 : 18
3958*l.* 10*s.* *Anſwer.*

3. Were there pence in the price, their parts might alſo eaſily be taken in the ſame method.

EXAMPLE.

℔. *s.* *d.*
5384 at 7 . 7½ *per* ℔.
3

1615 . 4
1*s.* · $\frac{1}{20}$ · · 269 . 4
6*d.* · $\frac{1}{2}$ · · 134 . 12
1½ · $\frac{1}{4}$ · · 33 . 13

Anſwer. 2052*l.* 13*s.*

4. Another ſhort and uſeful way, when the quantity is ſmall, is to multiply the price by it ; thus, if your quantity is expreſſed by one figure, it is done by one line.

EXAMPLE.

l. *s.* *d.*
5 *Yards of Velvet*, at 1 . 13 . 6 *per Yard.*
5

Product, 8*l.* 7*s.* 6*d.*

5. So any quantity that can be expreſſed by 2 figures, and ſome of 3, may be eaſily and ſpeedily done thus : Find out what two figures, multiplied together, will produce the given quantity, (as ſuppoſe 32 yards or pounds, &c. the given quantity which is produced by the multiplication of 8 by 4) and multiply the price by one of the ſaid figures ; and the product of that, multiplied again by the other, will give the anſwer.

EXAMPLES.

Yards. *s.* *d.*
32 at 15 . 8½ *per Yard.*
8

The va'ue of 8 *yards*, 6 . 5 . 8—
which multiplied by 4

gives 25*l.* 2*s.* 8*d. the price of* 32 *yards*,
—4 *times* 8 *being* 32.

Yards. *l.* *s.* *d.*
144 at 1 . 5 . 6 *per Yard.*
12

15 . 6 . 0
12

Product, 183*l.* 12*s.* 0*d.*

6. If no two figures will exactly produce thoſe given, take ſuch as will give the neareſt product to them, which will generally be within one or two, more or leſs; the value of which muſt accordingly be added or ſubtracted.

EXAMPLE.

℔. *l.* *s.* *d.*
55 at 1 . 10 . 9 *per* ℔.
9

The price of 9 ℔ 13 . 16 . 9
which multiplied by 6

gives the price of 54 ℔ 83 . 00 . 6
to which the price of 1 ℔. *added* 1 . 10 . 9

makes 84 . 11 . 3 *the value of* 55.

So, had it been multiplied by 7 and 8, which would have given the value of 56 ℔. too much by 1 ℔. the price of 1 ℔. muſt have been ſubtracted.

7. If the quantity has odd parts, it will be eaſy taking the value of ſuch parts from the given price: as in the following

EXAMPLE.

Yards.	*l.*	*s.*	*d.*
43½ at	2	12	7 *per Yard.*
			7
	18	8	1
			6
	110	8	6
qrs.	2	12	7
2½	1	6	3½
Product,	114*l.*	7*s.*	4½

TRETT *and* TARE.

TARE and TRETT are the allowances made to Merchants in buying their goods.

Tare, of what they can agree for *per* the whole, *per* cheſt, &*c.* or *per C.* for the weight of the bag, box, cheſt, &*c.* which contains the commodity.

Trett, for the waſte, motes, or duſt; and is always 4 ℔. *per* 104 ℔.

There is alſo another allowance ſometimes given of 2 ℔. for every 3 *C.* for the turn of the ſcale, called cloff, or clough.

Note, the whole weight, before any allowances are made, is called *groſs*; when part is deducted, the remainder is called *ſuttle*; but when all are taken from it, what is left, is called *neat.*

RULES.

RULES, with EXAMPLES.

1. The *Tare* of any quantity of goods may eaſily be found, if at ſo much in the whole, only by ſubtracting the ſaid allowance from the groſs weight; if at ſo much *per* cheſt, *&c.* by multiplying the pounds tare by the number of cheſts, *&c.* and ſubtracting as before; and if at ſo much *per C.* by taking ſuch part or parts of the groſs weight, as the allowance is of a *C.*

EXAMPLES.

Firſt ſort. Suppoſe 15 *C.* 2 *qrs.* 13 ℔. tare were allowed on 456 *C.* 1 *qr.* 19 ℔. of tobacco, what would be the neat weight;

	C.	*qrs.*	℔.	
From	456	1	19	*Groſs.*
Subtract	15	2	13	*Tare.*
Remainder	440	3	6	

Secona ſort. What's the neat weight of 3 frails of raiſins, each weighing 3 *C.* 2 *qrs.* 10 lb. groſs; tare at 20 lb. *per* frail?

	C.	*qrs.*	lb.		lb.
	3	2	10		20
			3	*Frails.*	3 *Frails.*
	10	3	2	*Groſs.*	60 lb. or 2 *qrs.* 4 lb.
		2	4	*Tare.*	
Anſwer,	10	0	26	*Neat.*	

		C.	*qrs.*	lb.		lb.
Third ſort.		246	3	12	*Groſs.*	*Tare* 14 *per C.*
	lb.					
	14$\frac{1}{8}$	30	3	12	*Tare.*	
Anſwer,		216	0	0	*Neat.*	

Again,

	lb.	C.	qrs.	lb.	lb.
Again,		364	1	19 *Groſs.*	*Tare* 16 *per C.*
	$16\frac{1}{7}$	52	0	$6\frac{1}{2}$ *Tare.*	
Anſwer,		312	1	$12\frac{1}{2}$ *Neat.*	

Note, 14 and 16 pounds may be called the ſtandards of Tare; for from them may any other number of pounds, more or leſs, be taken, as in the following

EXAMPLES.

	lb.	C.	qrs.	lb.	lb.
		573	2	13 *Groſs.*	*Tare* 17 *per C.*
	$14\frac{1}{8}$	71	2	$22\frac{1}{2}$	
	$2\frac{1}{7}$	10	0	27	
	$1\frac{1}{2}$	5	0	$13\frac{1}{2}$	
		87	0	7 *Tare.*	
Anſwer,		486	2	6 *Neat.*	

	lb.	C.	qrs.	lb.	lb.
		548	3	11 *Groſs.*	*Tare* 10 *per C.*
	$16\frac{1}{7}$	78	1	$17\frac{1}{2}$	
	$8\frac{1}{2}$	39	0	$22\frac{3}{4}$	
	$2\frac{1}{4}$	9	3	$5\frac{1}{2}$ *Tare.*	
		49	0	$0\frac{1}{4}$	
Anſwer,		499	3	$10\frac{3}{4}$ *Neat.*	

Note, 'Tis firſt found what Tare 16 lb. would produce; becauſe 8 lb. being the $\frac{1}{14}$ part of a *C.* would be a troubleſome Diviſor.

2. *Trett* being always 4 lb. *per* 104 lb. the conſtant method of finding it, is by taking the $\frac{1}{26}$ part of the line it is to be deducted from, 4 times 26 being 104. But it being difficult to divide by 26 in one line, it will be more ſafe and eaſy to do the work aſide, as in the firſt Example.

EXAMPLES.

lb.	C.	*qrs.*	lb.	
	428	1	19	*Groſs.*
$4\frac{1}{26}$	16	1	$25\frac{1}{2}$	*Trett.*
facit	431	3	$21\frac{1}{2}$	*Neat.*

lb.
Trett 4 *per* 104

```
26)428(16
   ---
   168
   ---
    12
     4
   ---
26)49(1
   --
    23
    28
   ---
   193
   193
    47
   ---
26)663(25
   ---
   143
   ---
    13
     4
   ---
26)52(2
   --
    0
```

	C.	qrs.	lb.	lb.
	836	2	17	*Groſs, Tare* 22 *per C,*
lb.				*Trett.* 4 lb. *per* 104 lb.
$16\frac{1}{7}$	119	2	$2\frac{1}{4}$	
$4\frac{1}{4}$	29	3	$14\frac{1}{2}$	
$2\frac{1}{2}$	14	3	$21\frac{1}{4}$	
	164	1	10	*Tare.*
	672	1	7	*Suttle.*
lb.				
$4\frac{1}{26}$	25	3	12	*Trett.*
facit	646	1	23	*Neat.*

3. *Clough* (which is always 2 lb. for 3 *C.*) may be found by taking the $\frac{1}{168}$ part of the line it is to be deducted from, 2 lb. being the $\frac{1}{168}$ part of 336 lb. or 3 *C.* But let the work be done aſide, as directed in finding the Trett. Or the allowance for *Clough* may be found, by firſt dividing the hundreds of the line it is to be taken from, by 3, which brings them into 3 C's; then 2 lb. being to be allowed for every 3 *C.* as many 3 *C*'s as it produces, ſo many 2 lb's it will allow; then dividing by 56 (the double pounds in a hundred) the quotient will be hundreds, and the remainder double pounds; to which adding, what may be allowed for the odd hundreds, quarters, and pounds of the given weight, it makes the whole *Clough*; which ſubtracted from the ſaid given weight, leaves the Neat. See both ways in the firſt Example.

EXAMPLE 1.

What will be the neat Weight of 5647 *C.* 3 *qrs.* 13 lb. *Groſs*, allowing for Clough 2 lb. for 3 *C?*

```
             C.    qrs.  lb.
            5647 . 3 . 13 Groſs.
              33 . 2 . 13 Clough.
            ---------------
            5614 . 1 . 00 Neat.
        C.
168)5647)33                 3)5647(1
    ----                      ----
    .607                   56)1882(33 . 2 . 12
    ----                      ----
     103                        202
       4                        ---
    ---- qrs.                14)34(2
 168)415(2                      --
    ----                        .6
     .79
      28
    ----
     635
     159
    ---- lb.
168)2225(13
    ----
    .545
    ----
     .41
```

Note, The remainder 34 (which are double pounds) is divided by 14, the double pounds in a *qr.* to bring the ſaid remainder into quarters of an hundred, making 2 *qrs.* and 12 lb. And 1 lb. being allowed for the odd *C.* 3 *qrs.* and 13 lb. the whole makes, as before, 33 *C.* 2 *qrs.* 13 lb.

EXAM-

EXAMPLE 2.

What's the neat weight of 3 hogſheads of tobacco, weighing, *viz.*

	Number.	C.	*qrs.*	lb.	
	1	5	3	18	
	2	4	2	11	
	3	5	1	19	
		15	3	20	*Groſs.*
lb.					
14⅛		1	3	27	
7½			3	27½	*Tare.*
		14	3	20½	*Suttle.*
4$\frac{1}{26}$			2	8¼	*Trett.*
		14	1	12¼	*Suttle.*
				9½	*Clough.*
Anſwer,		14	1	2¼	*Neat.*

Tare 7 lb. *per* C.
Trett 4lb. *per* 104lb.
Cough 2 lb. for 3 C.

C.
3)14(4
2

lb.
Note, The 4 three hundreds allow ——— 8
And the odd 2 C. 1 *qr.* 12 lb. ½ allow——— 1½
Total Clough 9½

EXAMPLE 1.

What will be the neat Weight of 5647 *C.* 3 *qrs.* 13 lb. *Groſs*, allowing for Clough 2 lb. for 3 *C*?

C.	*qrs.*	lb.	
5647	3	13	*Groſs.*
33	2	13	*Clough.*
5614	1	00	*Neat.*

C.
168)5647)33
.607
103
4
qrs.
168)415(2
.79
28
635
159
lb.
168) 2225(13
.545
.41

3)5647(1
56)1882(33 . 2 . 12
202
14)34(2
.6

Note, The remainder 34 (which are double pounds) is divided by 14, the double pounds in a *qr.* to bring the ſaid remainder into quarters of an hundred, making 2 *qrs.* and 12 lb. And 1 lb. being allowed for the odd *C.* 3 *qrs.* and 13 lb. the whole makes, as before, 33 *C.* 2 *qrs.* 13 lb.

EXAM-

EXAMPLE 2.

What's the neat weight of 3 hogſheads of tobacco, weighing, *viz.*

Number.	C.	*qrs.*	lb.	
1	5	3	18	
2	4	2	11	
3	5	1	19	
	15	3	20	*Groſs.*
lb.				
$14\frac{1}{8}$	1	3	27	
$7\frac{1}{2}$		3	$27\frac{1}{2}$	*Tare.*
	14	3	$20\frac{1}{2}$	*Suttle.*
$4\frac{1}{26}$		2	$8\frac{1}{4}$	*Trett.*
	14	1	$12\frac{1}{4}$	*Suttle.*
			$9\frac{1}{2}$	*Clough.*
Anſwer,	14	1	$2\frac{3}{4}$	*Neat.*

Tare 7 lb. *per* C.
Trett 4lb. *per* 104lb.
Cough 2 lb. for 3 C.

C.
3)14(4
2

	lb.
Note, The 4 three hundreds allow	8
And the odd 2 *C.* 1 *qr.* 12 lb. $\frac{1}{2}$ allow	$1\frac{1}{2}$
Total Clough	$9\frac{1}{2}$

CHAP. V.

Of INTEREST.

INTEREST is either { Simple, Or Compound, { which ariſes both from the Principal. { which ariſes only from Principal and Intereſt.

SIMPLE INTEREST.

SIMPLE INTEREST being one of the branches of Proportion, or the *Rule of Three*, is worked as follows:

1. The Intereſt of ony Sum for one year is found by this plain proportion, *viz.*

As 100*l.*
To the rate of Intereſt:
So is the given Principal,
To the Intereſt required.

EXAMPLE.

What's the Intereſt of 429 *l.* for a year, at 6*l. per Cent. per Annum?*

If

```
     l.            l.           l.
If 100 ——————————— 6 ——————————429
                                 6
                              ————
                             25|74
                                20
                              ————
                                20
                             14|80
                                12
                              ————
                              9|60
                                 4
                              ————
                              2|40
```

Anſwer, 25 *l.* 14 *s.* 9*d.*½

Note, The cutting off two figures to the right-hand, divides ſuch lines by 100. *See Abbreviations of Diviſion.*

2. To find the Intereſt of any ſum for 2 or more years, multiply the year's amount by the number of years required.

EXAMPLE.

What intereſt will the foregoing principal produce in 4 years, at the ſame rate of intereſt ?

```
        25  14  9½  the intereſt for one year.
                 4
        ——————————
Product 102 19  2
```

3. To find intereſt for months, divide the year's amount by the even parts they make of a year.

EXAMPLE.

What's the intereſt of 572 *l.* for 8 months, at 5*l.* *per Cent. per annum?*

l. *l.* *l.*
If 100——5——572
5
28|60
20
12|00

l. *s.*
28 . 12
M.
6½ 14 . 6
2⅓ 4 . 15 . 4
Anſwer, 19*l.* 1*s.* 4*d.*

4. If the time is weeks or days, inſtead of taking the even parts of a year as for months, it muſt be done by a ſecond ſtating in the *Rule of Three.*

EXAMPLES.

Ex. What's the intereſt of 321*l.* 16*s.* 8*d.* for 3 weeks, at 6*l.* *per Cent. per annum?*

l. *l.* *l.* *s.* *d.*
If 100——6——321 . 16 . 8
6
19|31 . 00 . 0
20
6|20
12
2|40
4
1|60

Anſwer 19*l.* 6*s.* 2*d.*¼ *for a year.*

Then,

Weeks. *l.* *s.* *d.* *Weeks.*
Then, *If* 52——19 . 6 . $2\frac{1}{4}$——3
3
l. *s.* *d.*
52)57 . 18 . $6\frac{3}{4}$(1 . 2 . $3\frac{1}{4}$
5
20
52)118(2
14
12
52)174(3
18
4
52)75(1
23

Ex. 2. What intereſt is now due on a bond of 420*l.* 10*s.* commencing intereſt at 5*l.* 10 *s. per Cent. per annum*, *July* the 10th, 1766; this being *March* the 25th, 1767?

l. *l.* *s.* *l.* *s.*
If 100——5 . 10——420 . 10
5
s. 2102 . 10
$10\frac{1}{2}$ 210 . 5
23|12 . 15
20
23 *l.* 2 *s.* 6 *d.* $\frac{1}{2}$ *for a Year.* 2|55
12
6|60
4
2|40

July	21
Auguſt	31
September	30
October	31
November	30
December	31
January	31
February	18
March	25
	258

Then,

Days. *l.* *s.* *d.* *Days.*

Then, *If* 365—23 . 2 . 6$\frac{1}{2}$———258

20

462

12

5550

4

22202

258

177616

111010

44404

4

365)5728116(15693(1

365

12)3923(11

2078

1825 2|0)32(6

2531 16*l.* 6*s.* 11*d.*$\frac{1}{4}$ *for* 258 *days.*

2190 23*l.* 2*s.* 6*d.* $\frac{1}{2}$ *for a year.*

.3411 39*l.* 9*s.* 5*d.*$\frac{3}{4}$ *total Intereſt.*

3285

.1266

1095

.171

Another way to caſt up Intereſt, *when the principal has odd money in it, and the time, weeks, or days.*

RULE.

Bring your principal into its loweſt denomination, and multiply it by the number of days; and if at 5 *l.*

per Cent. divide the product by 7300, the quotient gives the anſwer in the ſame denomination the principal was reduced into. *Note*, The number 7300 is found by multiplying 365 by 100, and dividing by 5.

EXAMPLES.

What's the intereſt of 432 *l.* 12 *s.* 6 *d.* for 8 days, at 5 *l.* *per Cent. per annum?*

```
           l.     s.    d.
          432  .  12  .  6
                  20
         --------
          8652
            12
       ----------
        103830
             8
       ---------- 12
73|00)830640(113
        73        ====
    --------          9 s. 5 d. ¾ Anſwer.
       100
        73
     ------
       276
       219
     ------
       5740
          4
73|00)229'60(3
       219
     ------
        10
```

If you would caſt up Intereſt at 6 *l.* *per Cent.* by this rule, you muſt add ⅕ of the quotient to the intereſt produced at 5 *l.* *per Cent.* and ſo at other rates, it is but adding or ſubtracting ſuch and ſuch parts of the quotient to or from what it produces at 5 *l.* For inſtance, ſuppoſe the laſt queſtion done the common way, for an

EXAMPLE.

```
        l.    s.
       420 . 10 Principal.
         20
       ----
       8410
        623 number of days.
      -----
      25230
     16820
    50460
   ------        s.   d.
73|00)52394|30(717 . 8¾
      511     s.
      ---    10 1/10 71 . 9¼
       129   ------------
        73       78|9 . 6
       ---
      ·564    l. 39|9 . 6   Anſwer, 1 far-
       511                  thing more than
      ----                  by the other way.
      ·5330
         12
      -----
73|00)639|60(8
      584
     ----
      ··5560
           4
      ------
73|00)222|40(3
      219
      ---
      ··340
```

Note, The Rate here being 5 *l.* 10 *s.* and 10 *s.* being $\frac{1}{10}$ of 5 *l.* there is therefore added $\frac{1}{10}$ of the quotient, or the amount, at 5 *l. per Cent.*

More Examples in Simple Intereſt.

At 6*l. per Cent.* what will 589 *l.* come to in a year?

Anſwer, 35 *l.* 6 *s.* 9 *d.*$\frac{1}{2}$.

What will the ſame ſum, at the ſame rate, amount to in 8 years?

Anſwer, 282 *l.* 14 *s.* 4 *d.*

What's the intereſt of 7020 *l.* for 5 months, at 5 *l. per Cent. per annum*?

Anſwer, 146 *l.* 5 *s.*

What's a week's intereſt of 1000 *l.* at 7 *l. per Cent*?

Anſwer, 1*l.* 6*s.* 11 *d.*

What will the intereſt of 5429 *l.* 10*s.* 8 *d. come* to in 20 days, at 5 *l. per Cent*?

Anſwer, 14*l.* 17*s.* 6*d.*

What intereſt is now due on a Navy-bill, commenceing intereſt at 6 *l. per Cent. per annum*, *June* the 4th, 1765, this being *January* the 10th, 1767, the principal being 836 *l*?

Anſwer, 80*l.* 7*s.* 10*d.*

COMPOUND INTEREST.

COMPOUND INTEREST, as was before ſaid, ariſes both from the principal and intereſt; (*i.e.*) when intereſt on money becoming due, and not paid, the ſame rate is allowed on the unpaid intereſt, as was before on the principal.

Therefore, to word ſums in this rule, after having found the firſt year's intereſt, as before, add it to the principal, and find the intereſt of the ſum; and ſo continue to add every year's produce, ſtill accounting the ſum a new principal.

EXAMPLE.

What will be the amount of 400 *l.* forborn 3 years and ½, at 6*l. per Cent. per annum*, compound Intereſt?

l. *l.* *l.*
If 100——6——400*l. Principal* 1*ſt year.*
6
l. 24|00

l. *l.* *l.*
If 100——6——424 *Principal* 2*d year.*
6
l. 25|44
20
s. 8|80
12
d. 9|60
4
2|40

If

l.
424
25 . 8 . 9$\frac{1}{2}$
———
If 100——6——449 . 8 . 9$\frac{1}{2}$ *Princip.* 3*d year.*
6
———
l. 26|96 . 12 . 9
20
———
s. 19|32
12
———
d. 3|93
4
———
3|72
l. *s.* *d.*
449 . 8 . 9$\frac{1}{2}$
26 . 19 . 3$\frac{3}{4}$
———
If 100 ——6——476 . 8 . 1$\frac{1}{4}$ *Princip.* 4*th year.*
6
———
l. *s.* *d.* *l.* 28|58 . 8 . 7$\frac{1}{2}$
28 11 8 20
M. ——— ———
6$\frac{1}{2}$ 14 5 10*s.* 11|68
476 8 1$\frac{1}{4}$ 12
——— ———
490 13 11$\frac{1}{4}$*d.* 8|23
Total amount. 4
———
94

Or, more plain :

Intereſt the 1ſt Year	24	0	0
2d ——	25	8	9½
3d ——	26	19	3¾
½ *Year*	14	5	10
Total Intereſt	90	13	11¼
Principal	400	0	0
Total amount	490	13	11¼

More EXAMPLES.

What will 1000 *l.* amount to in 3 years, at 6 *l. per Cent. per annum,* Compound Intereſt ?

Anſwer 1191*l.* 0*s.* 3*d.*¾

At 5*l.* 10*s.* *per Cent. per annum,* Compound Intereſt, what will 560*l.* 10*s.* 9*d.* amount to, forborn 5 Years and a half ?

Anſwer, 752*l.* 14*s.* 10*d.* ½.

CHAP. VI.

Of REBATE, *or* DISCOUNT.

REBATE, or DISCOUNT, is the abating ſo much money on a debt paid before it is due, as might be gained again by the money received, if put out to intereſt at the ſame rate, and for the ſame time. So 100*l.* preſent money would diſcharge a debt of 106 *l.* due at a year to come; rebate being made at 6*l.* *per Cent.* becauſe 100*l.* put out to intereſt for a year, at the ſaid rate, would regain the 6 *l.*

To work Sums in this RULE.

Firſt, find the intereſt of 100 *l.* for the time; then by a *Rule of Three* ſtating, of which the firſt number muſt be 100 *l.* with the intereſt found; the ſecond number, the intereſt alone (or 100 *l.* alone, if you would find the preſent Money); and the third, the debt, or Sum propounded, you will find the anſwer.

EXAMPLES.

Ex. 1. What's the rebate of 420 *l.* for a year, at 6 *l. per Cent. per annum?*

```
If 106——6——420
              6
            ——— l.
    106)2520(23
        212
        ———
        .400
         318
         ———
         .82
         ———
          20
         ——— s.
Anſwer, 23 l. 15 s. 5 d. ½   106)1640(15
         106
         ———
         .580
         .530
         ————
          .50
           12
          ——— d.
     106)600(5
         530
         ———
         .70
           4
         ———
     106)280(2
         212
         ———
         .68
```

Ex. 2. What preſent money will diſcharge a debt of 732 *l.* due at 3 months, rebate being made at 6 *l.* *per Cent. per annum?*

l. *l.*
6 *Intereſt for a year of* 100
Months—
3 $\frac{1}{4}$ 1 . 10
l. *s.* *l.*
Then, *If* 101 . 10 *fall to* 100, *what will* 732?
2 2

203 203)146400(721
1421
..430
406
.240
203
.37
20

Anſwer, 721 *l.* 3 *s.* 7 *d.* $\frac{1}{2}$ 203)740(3
609
131
12
203)1572(7
1421
.151
4
203)604(2
406
198 *Ex.*

Ex. 3. What rebate muſt be allowed on a Bill of Exchange for 85*l.* 10*s.* due *September* the 8th, this being *July* the 4th, rebate being made at 5 *l. per Cent. per annum?*

```
                        Days.       l.      Days.
                   If  365 ——————— 5 ——————— 66
July   27                                      5
Aug.   31                                   ————
Sept.   8                                    330
      ———                                     20
       66                                   ————  s.     d.
                                   365)6600(18  .  0¾
                                        365
                                       ————
                                       2950
                                       ————
                                       ..30
                                         12
                                       ————
                                        360
                                          4
                                       ————
                                   365)1440(3
                                       1095
                                       ————
                                        345
```

Then,

```
           l.    s.   d.    s.  d.    l.   s.
Then, If 100 . 18 . 0¾—18 . 0¾—85 . 10
          20              12         20
        ----            ----      ----
        2018             216      1710
          12               4        12
        ----            ----      ----
       24216             867     20520
           4                         4
       -----                    -----
       96867                     82080
                                   867
                               -------
                                574560
                               492480
                              656640
                             ---------  4
                      96867)71163360(734(2
                            678069    ----
                            ------ 12)183(3
                            .335646   ----
                             290601   15s. 3½
                            -------
                            .450450
Answer, 15s. 3d.½           387468
                            -------
                             .62982
```

Ex. 4. A Bill of Exchage for 250 *l.* is dated at *Amſterdam*, *June* the 14th, N. S. at uſance is accepted, and payment offered the 15th *ditto*, O. S. what muſt be then received, rebate being made at 6 *l. per Cent. per annum*?

Note, New Style is 11 days before Old Style.

Uſance ſignifies the uſual time of payment of bills between one country and another; which between *England* and *Holland* is one month; double Uſance, two months, &c. *Note alſo*, Merchants of *London* generally allow 3 days beyond the time appointed, calling them 3 days of grace. Therefore the bill being dated *June* the 14th N. S. or the 3d O S. and accepted at uſance, or a month, it becomes due *July* the 3d, O S. but 3 days of grace being allowed, makes it the 6th. So if the money is paid the 15th of *June* O. S. rebate muſt be made for 21 days. Therefore,

Days. *l.* *Days.*
If 365 —— 6 —— 21
6
126
20
s.
365)2520(6
2190
·330
12
d.
facit 6*s.* 10*d.* ¾ 365)3960(10
365
·310
4
365)1240(3
1095
·145

If

```
   l.   s.  d.       l.          l.
If 100 . 6 . 10¾ —— 100 ——————— 250
     20                          20
   ————                         ————
   2006                         5000
     12                           12
  ——————                      ——————
  24082                        60000
      4                            4
  ——————                      ——————  l.
  96331               96331)24000000(249
                            192662
                            ——————
                            .473380
                             385324
                             ——————
                             .880560
                              866979
                              ——————
                               .13581
                                   20
                               ——————  s.
                       96331)271620(2
                             192662
                             ——————
                              .78958
                                  12
                             ——————  d.
                       96331)947496(9
                             866979
                             ——————
Answer, 249 l. 2s. 9d. ¾      .80517
                                   4
                           ——————— far.
                       96331)322068 (3
                             288993
                             ——————
                             .33075
```

More

More EXAMPLES.

What's the rebate of 100 for $\frac{1}{2}$ a year, at 6 *l. per Cent. per annum?*

Anſwer, 2 *l.* 18*s.* 3*d.*

What's the rebate of 538 *l.* 10 *s.* for 5 months, at 5 *l. per Cent. per annum?*

Anſwer, 10 *l.* 19 *s.* 9 *d.* $\frac{1}{2}$

What preſent money is a debt of 629 *l.* due at 3 weeks, worth, rebate being made after the rate of 6 *l.* 10 *s. per Cent. per annum?*

Anſwer, 626 *l.* 16 *s.* 7 *d.* $\frac{1}{2}$

What preſent money will diſcharge 10000 *l.* due at 5 days, rebate being made at 8 *l. per Cent. per annum?*

Anſwer, 9989 *l.* 1 *s.* 5 *d.* $\frac{3}{4}$

CHAP.

CHAP. VII.

Of EQUATION *of* PAYMENTS.

THE deſign of this rule is, when ſeveral ſums are due at ſeveral times, to find a mean time for paying the whole debt.

The common way to do which is,

Multiply each ſum by its reſpective time, and add all the products together, dividing their total by the whole debt; the quotient is accounted the mean time of paying the whole debt.

EXAMPLE.

A owes to *B* 40 *l.* to pay at 3 months, 60 *l* at 5 months, and 100 *l.* at 10 months; at what time may the whole debt be paid together, without injury to either?

```
 l.     l.     l.     l.
 40     40     60    100
 60      3      5     10
100     ——     ——    ———
———    120    300   1000
200                  300
                     120
                    ————
            2|00)14|20(
                 ——————
```

Anſwer, 7 months.

Note, The times of payment, if not ſo given, muſt be reduced into one name.

But thoſe who are more exact take the pains to work theſe ſums by the following rule.

Firſt, they find the preſent worth of every particular ſum; then find in what time to come the ſum of theſe preſent worths will be increaſed to the total of all the particular ſums payable at the ſeveral times to come; and that is the true equated time for payment of the whole.

CHAP. VIII.

Of FELLOWSHIP, or COMPANY.

FELLOWSHIP, or COMPANY, is when two or more join their ſtocks, and trade together, dividing their gain or loſs proportionably.

FELLOWSHIP is either with, or without time. Queſtions without time, are worked by this proportion.

As the whole ſtock,
To the whole gain, or loſs :
So is each man's particular ſtock,
To his particular ſhare.

EXAMPLES.

A, *B*, and *C*, make a joint ſtock ; *A* puts in 460 *l.* *B* 510 *l.* and *C* 480 *l.* they gain 340 *l.* what part of it belongs to each ?

	l.
A,————	460
B,————	510
C,————	480
Total ſtock,	1450

```
1st. Then, If 145|0——340——46|0
                                        340
     l.    l.    l.                  ——————
2d. If 145|0—340—51|0                  1840
                 340                   138
             ———————                ——————— l.
                2040            145)15640(107
                 153                   145
             ——————— l.              ———————
          145)17340(119                .1140
                 145                    1015
             ———————                 ———————
                .284                    .125
                 145                     20
             ———————                 ——————— s.
                1390              145)2500(17
                1305                    145
             ———————                 ———————
                  85                    1050
                  20                    1015
             ——————— s.              ———————
          145)1700(11                   ..35
                 145                     12
             ———————                 ——————— d.
                .250                145)420(2
                 145                    290
             ———————                 ———————
                 105                    130
                  12                      4
             ——————— d.              ———————
          145)1260(8                145)520(3
                1160                    435
             ———————                 ———————
                .100                    .85
                   4
             ———————
          145)400(2
                 290
             ———————
                 110
```

l. —— l. —— l.

3d. And, If 145|0 ——— 340 ——— 48|0

340

1920

144

145)16320(112 l.

145

.182

145

.370

290

.80

20

145)1600(11 s.

145

.150

145

..5

12

60

4

145)240(1

145

.95

	l.	s.	d.	Rem.
A's *Share is*	107	17	2¾	85
B's *Share is*	119	11	8½	110
C's *Share is*	112	11	0¼	95
Value of the Remainder			½	

145)290(2

The whole gain	340	00	0	290

.0

And thus are theſe Sums proved

More EXAMPPES.

Three perſons make a joint ſtock ; *A* puts in 565 *l.* *B* 278 *l.* and *C* 629 *l.* they gain 1000 *l.* and what is each of their ſhares ?

		l.	*s.*	*d.*	*Remainders.*
Anſwer,	*A,*	383	16	7½	384
	B,	188	17	2	512
	C,	427	6	2¼	576

D, *E*, and *F*, trade together ; *D* puts in 230 *l.* *E* 529 *l.* and *F* 344 *l.* 10 *s.* their whole gain amounts to 520 *l.* what is that to each ?

		l.	*s.*	*d.*	*Remainders.*
Anſwer,	*D,*	108	7	7¼	271
	E,	249	5	7	841
	F,	162	6	9	1693

A, *B*, and *C*, make a bank of 3256 *l.* whereof *A* puts in 1026 *l.* *B* 985 *l.* and *C*, the reſt ; by misfortune they loſe 2000 *l.* what part of it muſt each bear ?

		l.	*s.*	*d.*	*Remainders,*
Anſwer,	*A,*	630	4	5	928
	B,	605	0	8¾	1240
	C,	764	14	10	1088

FELLOWSHIP *with* TIME.

FELLOWSHIP with TIME is thus worked ; Multiply each man's ſtock by the reſpective time he puts it in for, and add all the products, the total of which muſt be your firſt number through all the ſtatings ; the gain, or loſs, the ſecond (as before ;) and each man's particular ſtock, multiplied by its time, the third.

Note, All the particular times (if not ſo given) muſt be reduced into one name, (*i. e.*) all years, all months, all weeks, or all days, *&c.*

The reaſon of multiplying the ſeveral ſtocks by the times they are put in for, will appear from the following conſiderations. *Firſt*, Suppoſe *A* and *B* put into trade each 100 *l.* for 1 month, then certainly their gain, whatever it be, muſt be equally divided betwixt them Again, ſuppoſe *A* puts in 100 *l.* and *B* 200 *l.* both for the ſame time ; it is as plain that *B*'s ſhare of the gain muſt be twice as much as *A*'s, becauſe his ſtock is double : or if each put in 100 *l.* but for different times, *viz.* *A*, for one month, and *B*, for 2 ; then as *B*'s ſtock lies twice as long as *A*'s, ſo his gain, as before, muſt be twice as great. *Laſtly*, ſuppoſe *A* to put in 100 *l.* for one month, and *B* 200 *l.* for 2 months ; then ſince his ſtock is not only double, but the time he leaves it in double too, his ſhare of the profits muſt become 4 times greater than *A*'s. And ſo in all other caſes.

EXAMPLE.

A put into company 560 *l.* for 8 months, *B* 279 *l.* for 10 months, and *C* 735 *l.* for 6 months ; they gained 1000 *l.* what ſhare of it muſt each have ?

l.	*l.*	*l.*
560	279	735
8	10	6
A 4480	*B* 2790	C 4410
		2790
		4480
		11680

```
                l.         l.      l
1ſt. Then, If 1168|0 ——1000—448|0
                              1000
                           ———————— l.
                      1168)448000(383
                           3504
                           ——————
                           .9760
                            9344
                           ——————
                            .4160
                             3504
                            —————
                             .656
                               20
                            ————— s.
                      1168)13120(11
                           1168
                           —————
                           .1440
                            1168
                           —————
                            .272
                              12
                           ————— d.
                      1168)3264(2
                           2336
                           —————
                           .928
                              4
                           —————
                      116 )3712(3
                           3504
                           ————
                           .208
```

2d.

```
            l.          l.        l.
2d. If 1168|0——1000——279|0
                         1000
                       ———— l.
               1168)279000(238
                       2336.
                       ————
                       .4540
                        3504
                       ————
                       10360
                        9344
                       ————
                        1016
                          20
                       ———— s.
                1168)20320(17
                        1168
                       ————
                       .8640
                        8176
                       ————
                        .464
                          12
                       ———— d.
                  1168)5568(4
                        4672
                       ————
                        .896
                           4
                       ————
                  1168)3584(3
                        3504
                       ————
                        ..80
```

l. *l.* *l.*
3*d.* *If* 1168|0 —— 1000 —— 441|0
1000

l.
1168)441000(377
3504
.9060
8176
.8840
8176
.664
20

s.
1168)13280(11
1168
.1600
1168
.432
12

d.
1168)5184(4
4672
.512
4
1168)2048(1
1168
.880

	l.	*s.*	*d.*	*Rem.*
A's *Share is*	383	11	2¾	208
B's *Share is*	238	17	4¾	80
C's *Share is*	377	11	4¼	880

Value of the Remainders ¼ 1168)1168(1
1000 . 00 . 0 —— ...0
The whole gain.

Mere

More EXAMPLES.

A, *B*, and *C*, agreeing to trade together, *A* puts in 529 *l.* for 4 months; *B* 329 *l.* for 7 months; and *C* 900 *l* for 2 months; by their trade they gain 540 *l.* What part of it belongs to each?

		l.	*s.*	*d.*	*Remainders.*
Anſwer,	*A,*	183	14	8	2304
	B,	199	19	5	1222
	C,	156	5	10¾	2583

Theee perſons make a joint ſtock; *A* puts in 400 pieces of holland, each containing 96 yards, at 5*s.* 6*d.* *per* ell *Flemiſh* for 5 months; *B* 600 guineas for 8 months; and *C* 1800 *l.* for 4 months, but at 3 months end takes out 800 *l.* They gain in all 1000 *l.* how much is each of their ſhares?

		l.	*s.*	*d.*	*Remainders.*
Anſwer,	*A,*	860	4	3½	3996
	B,	61	11	8	1920
	C,	78	4	0¼	2568

CHAP. IX.

Of ALLIGATION.

ALLIGATION is the compounding many ſimples into one maſs, according to any required price, or proportion.

This rule is uſually divided into two parts, diſtinguiſhed by the names of *Medial* and *Alternate*.

ALLIGATION MEDIAL.

ALligation Medial anſwers ſuch queſtions as, having the quantities and prices of the ſeveral ſimples given, only demand the mean rate of any particular part of the compoſition.

RULE.

As the ſum of all the ſimples,
To their total value:
So is any quantity of the mixture,
To the price thereof.

EXAMPLES.

(*Ex. 1ſt.*) A grocer would mix 18 ℔. of currants at 5*d per* ℔. and 29 ℔. at 6*d. per* ℔. with 12 ℔. at 8 *d. per* ℔. at what rate may he ſell a pound of the mixture?

Firſt, place the quantities and their values, as below.

℔.	d.			s.	d.
18	*at* 5	*per* ℔. *comes to*		7	6
29	*at* 6	——— *to*		14	6
12	*at* 8	——— *to*		8	0
Sum of all the Sim. 59			£. 1	10	0 *tot. val.*

l. *s.*
Then, *If* 59 *comes to* 1 . 10 *what will* 1 ?
20
—
30
12
— *d.*
59)360(6 *Anſwer,*
354
—
. 6

Ex. 2. A grocer would with 12 *C.* of ſugar, at 3 *l. per C.* mix 3 *C.* at 50 *s. per C.* and 8 *C.* at 45 *s. per C.* What will 5 *C.* of this mixture be worth?

The quantities and their values, placed as before, will ſtand thus:

C.		*l.*	*s.*
12	*at* 3 *l. per C. is*	36	00
3	*at* 50 *s. per C. is*	7	10
8	*at* 45 *s. per C. is*	18	00
Sum of the Simples 23		61	10 *tot. val.*

Then,

```
           C.         l.  s.        C.
Then, If 23 ———————— 61 . 10 ——————— 5
                     20
                   ————
                   1230
                      5
                   ————
                 23)6150(26|7
                    46   ——
                 ——————— 13 l. 7 s. 4 d. ½ facit.
                    155
                    138
                   ————
                   .170
                    161
                   ————
                    ..9
                     12
                   ————
                 23)108(4
                     92
                   ————
                    .16
                      4
                   ————
                  23)64(2
                     46
                   ————
                     18
```

The proof of this is as in the *Rule of Three*, only by reverſing the proportion.

So the firſt example may be proved by this backſtating
℔. *d.* ℔.
If 1————6————59

And the laſt by this;
C. *l.* *s.* *d.* *C.*
If 5——13 . 7 . 4$\frac{1}{2}$——23

remembering in both to take in the remainders.

More EXAMPLES.

If a vintner mingleth 13 gallons of canary, at 6*s*. 8*d*. *per* gallon, with 20 gallons of white-wine, at 5*s*. *per* gallon, and 10 gallons of cyder, at 3*s*. *per* gallon, at what rate muſt he ſell a quart of the mixture?

Anſwer, 1*s*. 3*d*.

If with 40 buſhels of corn, at 4*s*. *per* buſhel, there are mixed 10 buſhels at 6*s*. *per* buſhel; 30 buſhels at 5*s*. *per* buſhel, and 20 buſhels at 3*s*. *per* buſhel, what will 10 buſhels of the miſtling be worth?

Anſwer, 2 *l*. 3 *s*.

ALLIGATION ALTERNATE.

ALligation Alternate teacheth what quantities of ſeveral ſimples (whoſe values are given) muſt be mixed, that the compoſition may bear a price propounded.

This ſort of *Alligation* has three varieties.

VARIETY 1.

When the price of each ſimple is expreſſed, but no quantity given, and it is required how much of each ſimple muſt be mixed, to ſell any part of the compoſition at a mean price propounded?

In this variety you have only to link the ſeveral extremes rightly together, and to find the true differences betwixt them and the mean; for thoſe differences are the quantities ſought.

EXAMPLE.

A vinter would mix four ſorts of wine, of 12 *d*. of 18 *d*. of 24*d*. and of 26*d*. *per* quart; what quantity of each muſt he take to ſell the mixture at 20*d*. *per* quart?

To place your numbers, and link the extremes, obſerve theſe

RULES.

1*ſt*. Set down the ſeveral given prices (which muſt always be of one name) in a column, one under another; then, towards your left hand, draw a line of connection; and on the other ſide of the ſaid line place the mean rate.

Thus then will ſtand the given numbers of the given example:

$$20\left\{\begin{array}{l}12\\18\\24\\26\end{array}\right.$$

2*d*. Link them two and two together, always obſerving to join a greater and a leſs than the mean; and againſt each extreme place the difference betwixt the mean and its yoke-fellow:

Thus,

$$20\left\{\begin{array}{l|l}12 & 4\\18 & 6\\24 & 8\\26 & 2\end{array}\right.$$

So the difference betwixt the extreme 12, and 20 the mean, being 8, is placed againſt 24, its yoke-fellow; and between 18 and 20 being 2, is placed againſt 26: ſo alſo the difference betwixt 24 and 20 being 4, is placed againſt 12; and 6, the difference betwixt 26 and 20, is ſet againſt 18.

Therefore, if he mixes 4 quarts at 12 *d.* 6 at 18 *d.* 8 at 24 *d* and 2 at 26 *d. per* quart, he may ſell the mixture at 20 *d. per* quart.

The proof of which is eaſy. For the ſum of the differences valued at the mean rate, will be found equal to the amount of the particular differences at their given prices.

For 20 quarts, the ſum of the differences at 20 *d.* *per* quart, is 400 pence; and 4, 6, 8, 2 quarts, the particular differences, at 12, 18, 24, 26 pence *per* quart, will come to the ſame ſum.

But note, as many different ways as the extremes may be combined, ſo many different anſwers may be given to the queſtion, yet all true.

So the ſame example being combined, thus:

$$20\left\{\begin{array}{l|l}12 & 6\\ 18 & 4\\ 24 & 2\\ 26 & 8\end{array}\right\}\textit{Anſw.}\quad 20.\left\{\begin{array}{l|l|l}12 & 4.6 & 10\\ 18 & 4.6 & 10\\ 24 & 8.2 & 10\\ 26 & 8.2 & 10\end{array}\right\}\textit{Anſw.}$$

gives thoſe different, but true anſwers, as may be proved by the ſame rule.

In the laſt of theſe examples the numbers being doubly combined (*i. e.*) each with two others, make each have two differences ſet againſt it; for 12, the firſt extreme, being linked both with 24 and 26, muſt have both their differences, *viz.* 4 and 6, placed againſt it; and ſo the reſt. Which double difference muſt be added, and placed as above.

But when in a queſtion there is but one extreme leſs, or but one greater than the mean; ſuch a queſtion will admit of but one anſwer, that ſingle extreme being to be linked with all the reſt.

EXAMPLE.

A grocer would mix ſugars, at 5 *d.* 6 *d.* and 10 *d.* *per* ℔. ſo as to ſell the compound at 8 *d.* *per* ℔. what quantity of each muſt he take?

$$8\left\{\begin{array}{l|l|l}5 & 2 & 2\\ 6 & 2 & 2\\ 10 & 3.2 & 5\end{array}\right.$$

The numbers being placed, linked, and differenced, (as above) according to the foregoing rules, it appears, to ſell his ſugar at 8 *d. per* ℔ he muſt mix 2 ℔. at 5 *d.* 2 ℔. at 6 *d.* and 5 ℔. at 10 *d. per* ℔.

The reaſon of theſe combinations, and the alternate placing of their differences, will appear from this plain conſideration, *viz.* that thereby whatever is loſt upon the quantity ſold, whoſe given price exceeds the mean, is gained again upon the quantity, whoſe given price is leſs than the mean.

More *EXAMPLES.*

A tobacconiſt wou'd mix tobacco of 2*s.* and 1*s.* 6*d.* and 1*s.* 3*d.* *per* ℔. ſo as to ſell the compound at 1*s.* 8*d.* *per* ℔. what quantity of each muſt he take?

	℔.		*s.*	*d.*	
Anſwer,	7	at	2	0	*per* ℔.
	4	at	1	6	
	4	at	1	3	

Several ſorts of tea, *viz.* of 13*s.* 18*s.* 26*s.* and 30*s.* *per* ℔. are to be ſo mixed, that the compound may be worth 22*s.* *per* ℔. what quantity of each muſt there be?

	℔.		*s.*	
Anſwer,	8	at	13	*per* ℔.
	4	at	18	
	4	at	26	
	9	at	30	

VARIETY 2.

Here we have the price of all the ſimples, and the quantity of one given, to find the ſeveral quantities of the reſt: ſo that one meaſure or quantity, may be ſold at a mean rate propounded.

When the numbers have been ſet down, and their ſeveral differences found by the foregoing rule, you muſt proceed by this proportion.

As the difference of that number whoſe quantity is given,

To the reſt of the differences one after another:

So the quantity given,

To the ſeveral quantities ſought.

EXAMPLE.

A diſtiller would with 40 gallons of *French* brandy, at 12*s. per* gallon, mix *Engliſh* brandy at 7*s. per* gallon, and ſpirits at 4*s. per* gallon: how much of the laſt two ſorts muſt be added, to fill the whole at 8*s. per* gallon?

$$8\left\{\begin{array}{l|l|l} 12 & 1\cdot 4 & 5 \\ 7 & 4\cdot & 4 \\ 4 & 4\cdot & 4 \end{array}\right.$$

Now, it is plain, that were there but 5 gallons of the *French* brandy, there muſt alſo be but 4 gallons of each of the other liquors; but ſince there are to be 40 gallons of the *French* brandy, ſay,

As 5, the difference againſt its price,
To 4, the difference againſt the price of the *Engliſh* brandy,
So is 40, the given quantity of the *French*,
To the quantity of the *Engliſh* ſought:
Which by this proportion ſhould be found to be 32.
Then again,
As the firſt difference 5,
To 4, the difference againſt the price of the ſpirits:
So 40, the ſaid given quantity,
To the quantity of ſpirits ſought.
Which becauſe their differences are alike, will be the ſame as of the *Engliſh* brandy, *viz.* 32.
This variety has the ſame proof as the former.

More EXAMPLES.

How much tea at 14*s. per* ℔. and of 18*s. per* ℔. muſt be mixed with 8 ℔. at 24*s. per* ℔. ſo as to allow the ℔. at 20*s*?

$$\textit{Anſwer,}\left\{\begin{array}{l} 8 \text{ ℔. at } 24\,s, \\ 4 \text{ ℔. at } 18\,s, \\ 4 \text{ ℔. at } 14\,s. \end{array}\right.$$

I would with 21 ℔. of ſnuff, at 8*s. per* ℔. mix ſo much of 4*s. per* ℔. and ſo much of 5*s. per* ℔. that I might afford to ſell the mixture at 6 *s. per* ℔. what quantity of each muſt I take?

$$\textit{Anſwer,}\left\{\begin{array}{l}21 \text{ at } 8\\ 14 \text{ at } 4\\ 14 \text{ at } 5\end{array}\right\}\textit{per}\text{ lb.}$$

VARIETY 3.

In this variety the particular prices of each ſimple being given, as alſo the mean rate of the compound, it is required how much of each ſort muſt be taken, to make up the quantity propounded?

Which (after the work of the firſt variety) is performed by this proportion;

As the ſum of the differences.
To the total quantity:
So each particular difference,
To its particular quantity.

EXAMPLE.

A druggiſt hath 4 ſorts of green tea, at 5*s.* 6*s.* 8*s.* and 9*s. per* lb. and would make up a quantity of 87 lb. to ſell at 7*s. per* lb. how much of each muſt he take?

See the work.

$$7\left\{\begin{array}{l}5\\ 6\\ 8\\ 9\end{array}\right. \begin{array}{l}\text{lb.}\qquad\quad s.\\ 2\text{—}29 \text{ at } 5 \textit{ per} \text{ lb.}\\ 1\text{—}14\tfrac{1}{2} \text{ at } 6 \textit{ per} \text{ lb.}\\ 1\text{—}14\tfrac{1}{2} \text{ at } 8 \textit{ per} \text{ lb.}\\ 2\text{—}29 \text{ at } 9 \textit{ per} \text{ lb.}\\ \overline{6}\end{array}$$

lb. lb. lb.
Iſ 6——87——2
2
—— lb.
6)174(29
——
0

lb. lb. lb.
Iſ 6——87——1
1
—— lb.
6)87(14½
——
3

More

More EXAMPLES.

A vintner hath 3 ſorts of wine, of 6*s.* 8*s.* and 10*s.* *per* gallon, of all which he would compoſe a quantity of 120 gallons, to ſell at 7*s.* *per* gallon: what quantity muſt there be of each?

$$\begin{array}{r} \\ \textit{Anſwer,} \end{array}\left\{\begin{array}{rcl} \textit{gall.} & & \textit{s.} \\ 80 & \textit{at} & 06 \\ 20 & \textit{at} & 08 \\ 20 & \textit{at} & 10 \end{array}\right\} \textit{per}\ \text{Gall.}$$

A grocer having ſugars of 5*d.* 6*d.* 8*d.* 10*d.* and 11 *d.* *per* lb. would make a mixture of 3 *C. wt.* to ſell at 7*d.* *per* pound: how much of each muſt he take?

$$\textit{Anſwer,}\left\{\begin{array}{rcl} \text{lb.} & & \textit{d.} \\ 28 & \textit{at} & 05 \\ 196 & \textit{at} & 06 \\ 56 & \textit{at} & 08 \\ 28 & \textit{at} & 10 \\ 28 & \textit{at} & 11 \end{array}\right\} \textit{per}\ \text{lb.}$$

CHAP. X.

Of POSITION, *or the Rule of* FALSE.

THIS rule hath not the name of *Falſe*, from its being in itſelf really erroneous, but only becauſe that by the help of falſe ſuppoſed numbers we find the truth.

It is uſually divided into two parts, *ſingle* and *double*.

Single POSITION.

IN *Single Poſition* we make but one ſuppoſition, with which working as with a true number, we regulate the reſult by this proportion, *viz.*

As the total ariſing from the error,
To the true total:
So the ſuppoſed part,
To the true one.

EXAMPLES.

1*ſt*. *A*, *B*, and *C*, deſigning to buy a quantity of lead, to the value of 140 *l.* agree that *B* ſhall pay as much again as *A*, and *C* as much again as *B*, what then muſt each pay?

Now ſuppoſe *A* to pay 10 *l.* then *B* muſt pay 20 *l.* and *C* 40 *l.* the total of which is 70 *l.* but ſhould be 140 *l.* Therefore,

If 70l. ſhould be 140 *l.* what ſhould 10 *l.* *be?*

	l.	
Anſwer,	20	*for* A's *ſhare;*
which doubled, makes	40	*for* B's *ſhare;*
and that again doubled, gives	80	*for* C's *ſhare;*
The total of which is	140	

2. Suppoſe the ſum of 20 *l.* were to be paid by *A*, *B*, and *C*, thus, $A\frac{1}{2}$, $B\frac{1}{3}$, $C\frac{1}{4}$; what muſt each pay according to that proportion;

	l.	*s.*	*d.*
Firſt, the $\frac{1}{2}$ *of* 20 *l. being*	10	00	0
the $\frac{1}{3}$ ————	6	13	4
and the $\frac{1}{4}$ ————	5	00	0
The total will be	21	13	4

But it ſhould have been but 20l. *therefore the parts found ſhould be thus regulated:*

$$\begin{array}{ccccc} l. & s. & d. & & l. \\ \textit{If } 21 & 13 & 4 & \textit{fall to} & 20, \\ & \textit{what will} & & & \end{array} \left\{\begin{array}{c} l.\ \ s.\ \ d. \\ 10.00.0 \\ 06.13.4 \\ 05.00.0 \end{array}\right\} \textit{Anſwer} \left\{\begin{array}{c} l.\ \ s.\ \ d. \\ 9.04.7\frac{1}{4} \\ 6.03.0\frac{3}{4} \\ 4.12.3\frac{1}{2} \end{array}\right.$$

which, with the addition of $\frac{1}{2}$, *makes juſt* 20.00.0-

$$\textit{found by dividing the total of the remainders} \text{——} \left\{\begin{array}{l} 28 \\ 36 \\ 40 \end{array}\right.$$

$$\textit{by the common diviſor } 52)104(2$$

$$.0$$

More EXAMPLES.

A gentleman bought a chaiſe, horſe, and harneſs, for 50 *l.* the horſe come to twice the price of the harneſs, and the chaiſe to twice as much as both horſe and harneſs; what did he give for each?

		l.	*s.*	*d.*	*Remainders.*
Anſwer,	*the harneſs*	05	11	$1\frac{1}{4}$	——— 15
	the horſe	11	02	$2\frac{1}{2}$	——— 30
	the chaiſe	33	06	8	———

A, *B*, and *C*, diſputing of their age, *A* affirms he is as old as *B*, and $\frac{1}{2}$ as old as *C*, and *B* ſays, that he is $\frac{3}{4}$ as old as *C*, upon which *C* ſays, he is ſure both their ages, added to his, will make 110 years. I demand the age of each.

Anſwer	*A*,	$45\frac{5}{6}$
	B,	$27\frac{3}{6}$
	C,	$36\frac{4}{6}$

Double POSITION.

IN *Double* POSITION two ſuppoſitions are required, which (if both prove falſe) are, with their errors, to be thus diſpoſed.

Set down both your ſuppoſitions, and againſt each of them their reſpective errors, marked thus +, if too much; or thus —, if too little.

Then multiplying them croſs-ways, that is, the firſt ſuppoſition by the ſecond error, and the ſecond ſuppoſition by the firſt error, if both the errors are alike, *i. e.* both too little, or both too much, ſubtract the leſſer product from the greater, and divide the remainder by the difference of the errors. But if the errors are unlike, *i. e.* one too little, and the other too much, the ſum of theſe products muſt be divided by the ſum of the errors: the quotient will give the true number, exactly anſwering all the demands of the queſtion.

Or the ſuppoſitions and their errors being placed as before, work by this proportion as a general rule, *viz.*

As the difference of the errors, if alike,
Or their ſum, if unlike,
To the difference of the ſuppoſitions:
So either error to a fourth number,
which accordingly added to, or ſubtracted from the ſuppoſition againſt it, will anſwer the queſtion.

EXAMPLES.

1*ſt*. *A*, *B*, and *C*, playing at cards, the money ſtak'd was 324 crowns; but diſagreeing, each ſeized as many as he could; *A* got a certain number, *B* as many as *A*, and 15 over; *C* got a fifth part of both their ſums added together: how many had each?

First, *ſuppoſe* A *got* 50
then B'*s ſhare will be* 65
and C'*s* ——————— 23

All which added, make but 138, *which is* 186 *too few*.
Then, *ſuppoſing again* that A *got* 80
B'*s ſhare will be* 95
and C'*s* ——— 35

The total of which is ſtill but 210, *too few by* 114.

Therefore, ſetting down the ſuppoſitions, and their errors (mark'd as before directed) againſt them, they will ſtand thus:

Suppoſitions.	*Errors.*
50	186
80	114

then multiplied croſs-ways, the products will be 14880, and 5700; the leſſer of which (the errors being alike, *i. e.* both too little) being taken from the greater, there will remain 9180 for a dividend; which divided by 72, the difference of the two errors, qotes 127½, the true number of crowns *A* took up; as will appear, by adding 15 to them for *B's* ſhare, and dividing the ſum of *A's* and *B's* by 5, for *C's* ſhare; for ſo their particular number will be found to be as follow:

A's———$127\frac{1}{2}$
B's———$142\frac{1}{2}$
C's——— 54

The total of which is 324, *the number of crowns given.*

Or by the other rule;

As 72, the difference of the errors (becauſe alike) to 30, the difference of the ſuppoſitions:

$$\textit{So is}\left\{\begin{matrix}186\\ \textit{or}\\ 114\end{matrix}\right\}\textit{to}\left\{\begin{matrix}77\frac{1}{2}\\ \\ 47\frac{1}{2}\end{matrix}\right\}\begin{matrix}\textit{which}\\ \textit{added}\\ \textit{to}\end{matrix}\left\{\begin{matrix}50\\ \textit{or}\\ 80\end{matrix}\right\}\textit{gives } 127\tfrac{1}{2}\textit{, as before.}$$

EXAMPLE 2.

When firſt the marriage-knot was ty'd
Betwixt my wife and me,
My age did her's as far exceed,
As three times three does three.
But after ten and half ten years,
We man and wife had been,
Her age came up as near to mine,
As eight is to ſixteen.
—————————Now, pray,
What were our ages on the wedding-day?

First, *ſuppoſe her* 21 *then he muſt be* 63
then adding to each 15 ——————— 15

her age becomes 36 *his* 78

This error therefore is 3 years too few.

But ſuppoſing her 9, the error will prove 3 too many.

Setting down therefore againſt the ſuppoſitions the errors, with their reſpective marks, they will ſtand thus:

Sup-

Suppoſitions.		*Errors.*
21	—	3
9	+	3

Then multiplying croſs-ways, 3 times 21 is 63
3 times 9 is 27

Theſe products (becauſe unlike) added, make 90 for a dividend, which divided by 6, the ſum of the errors, quotes 15, her true age; proved thus:

She being 15, *he muſt be* —— 45
15 *years added to each* 15
Make her 30, *juſt half of his* —— 60

Or, by the other rule:

As 6, the ſum of the errors, (becauſe unlike) to 12, the difference of the ſuppoſitions:

So is the common error, 3 to 6; which either ſubtracted from 21, or added to 9, gives 15, as before) for her age.

More EXAMPLES.

What number is that to which if you add $\frac{1}{4}$ of itſelf, and from the ſum ſubtract $\frac{1}{3}$ of itſelf, the remainder will be 210?

Anſwer, 229.

A gentleman caught a fiſh, whoſe head was 6 inches long, the tail as long as the head, and half as long as the body; the body was juſt the length of the head and tail: I demand the length of the fiſh?

Anſwer, 48 inches, or 4 foot.

CHAP. XI.

Of EXCHANGE.

EXCHANGE is the receiving money in one country, for the value paid in another.

NOTE.

The par of exchange is certain and fix'd, being always *par pro pari, like for like,* according to the weight and fineneſs of the coin.

But the courſe or current running price betwixt any two countries, riſes and falls upon every occaſion.

It would be both needleſs and endleſs to write of every kind of exchange; I ſhall therefore only give examples of the exchange of *England*, with ſome other chief countries of *Europe*. And firſt, with

FRANCE.

At *Paris*, *Lyons*, *Rouen*, &c. they keep their accounts in livres, ſols, and deniers; and exchange upon the crown, the par of which, in ſterling money is 4*s.* 6*d.*

NOTE.

NOTE.

12 Deniers	make	1 Sol.
20 Sols		1 Livre.
3 Livres		1 Crown.

French money is changed into *Sterling* by this proportion;

As a Crown to the given rate:
So is the given *French* ſum to the *Sterling* ſought.

Or,

By ſuppoſing the given *French* crowns, *goods*, and the rate of exchange, the *price*; ſo caſting up their value by *practice.*

EXAMPLES.

1*ſt*. What *ſterling* money muſt be paid in *London*, to receive in *Paris* 438 crowns; exchange at 56 *d.* *per* Crown?

```
   Cr.    d.     Cr.            Cr.   d.    s. d.
If 1 —— 56 —— 438           438 at 56 or 4  8 per Cr.
                56           ————————————
          ———————— 4 s. 1/5   87 . 12
              2628 8 d. 1/6   14 . 12
             2190             ——————————
          ————————           102 .  4
         12)24528(
          ————————
         2|0)204|4
          ————————
           102 l. 4 s.
```

2*d*. Change 537 crowns, 14 ſols, 10 deniers, into *Sterling* money; exchange at 52 *d*.½ *per* crown.

```
  Cr.  d.    Cr.  Sols. De.                Cr.  Sols. De.    d.
If 1—52½—537 . 14 . 10                    537 . 14 . 10 at 52½
   60   2      60                         ———
   12———   ————————              4s. 1/5  107 .  8
  ———105    32234                4d. 1/12   . 8 . 19
   720         12                  ½  1/8   . 1 . 2 . 4½
           ————————      Sols 10      1/6   . . .   . . 8¾
            386818            2       1/5   . . .   . . 1¾
               105            2       1/5   . . .   . . 1¾
         —————————       Den. 10      1/12  of 10 Sols.  ½
          1934090                           ———————————
         3868180                            117 l. 10s. 5d.¼ fac.
         ————————      2
72|0)4061589|0(56410
       360        ————————
       ———      12)28205(5
       .461       ————————
        432       2|0)235|0
       ———        ————————
       .295       117l. 10s. 5d.¼ facit.
        288
       ———
         .78
         72
        ———
          69
           2
        ———
     72)138(1
          72
        ———
          66
```

Sterling money is changed into *French* by this proportion :

As the rate of exchange, to one crown :
So is the given *Engliſh* ſum, to the *French* ſought.

For EXAMPLES.

Take the reverſe of the former queſtions ; which will alſo be their proof.

1*ſt.* How many *French* crowns muſt be paid in *Paris*, to receive in *London* 102*l.* 4*s.* *Sterling* ; exchange at 56 *d. per* crown ?

```
    d.        Cr.          l.    s.
If 56 ———— 1 ———— 102 . 4
                    20
                   ————
                   2044
                     12
                ————Crowns.
             56)24528(438
                224
                ———
                 212
                 168
                 ———
                  448
                  448
                  ———
                    0
```

2*d*. Change 117*l*. 10*s*. 5*d*.$\frac{1}{4}$ *Sterling* into *French* crowns; exchange at 52 *d*.$\frac{1}{2}$ *per* crown.

d. *Crown*. *l*. *s*. *d*.
If $52\frac{1}{2}$ —— 1 —— 117 . 10 . $5\frac{1}{4}$
4 20
——— ———
210 2350
12
———
28205
4
——— *Crowns*. *Sols*. *Deniers*.
21|0)11282|1(537 . 14 . 10
105
———
.78
63
———
152
147
———
51

51
60
—— *Sols*.
21|0)306|0(14
21
———
.96
84
———
12
12
———
144
66 *the remainder of the other ſum*.
———
21)210(10 *deniers*.
21
——
.0

More EXAMPLES.

If I pay in *Paris* 8297 *crowns*, 13 *ſols*, 8 *deniers*, what may I draw my bill for to *London*, when the exchange is at 54*d*.$\frac{1}{4}$ *per* crown?

Anſwer, 1875*l*. 10*s*. 4*d*.$\frac{1}{2}$.

What *Sterling* money is equal to 954 *crowns*, 52 *ſols*, when the exchange is at 58*d*.$\frac{1}{8}$ *per* crown?

Anſwer, 231*l*. 5*s*. 1*d*.$\frac{1}{2}$.

If I pay in *London* 536*l.* what *French* money ſhall I receive in *Paris* exchange at 57 *d. per* crown ?

Anſwer, 2256 *crowns*, 50 *ſols*, 6 *deniers*.

What *French* money is equal to 1529 *l.* 10 *s.* exchange at 53 *d.*¼ *per* crown ?

Anſwer, 6893 *crowns*, 31 *ſols*, 3 *deniers*.

Secondly, with ITALY.

IN *Genoa*, *Leghorn*, *&c.* they keep their accompts in livres, ſols, and deniers ; and exchange upon the piece of eight, or dollar ; the *par* of which with *London*, is alſo 4*s.* 6*d.* *Sterling*.

NOTE.

At *Genoa* 5 livres make a piece of eight, at *Leghorn* 6. Obſerve the rules for exchanging with *France*.

EXAMPLES.

How much *Sterling* ſhall I receive in *London*, if I pay in *Genoa* 820 dollars ; exchange at 51*d. per* dollar, or piece of eight.

```
Dollar.  d.   Dollars.        Dollars.
If 1 ——51 ——820              820 at 51d. per dollar.
             51                4
           ————              ————
            820   d.         3280
           4100   3¼          205
           ————              ————
       12)41820(            348|5
           ————              ————
       2|0)348|5            174l. 5s.
           ————
          174l. 5s.
```

Or back again;

If in *London* I pay 174*l.* 5*s.* *Sterling*, how many dollars ſhall I receive in *Genoa*; exchange at 51*d.* *per* dollar?

d. *Dol.* *l.* *s.*

If 51 —— 1 —— 174 . 5

20

3485

12

---

51)41820(820

408

. 102

102

. . . 0

Venice exchanges by the ducat; the *par* about 56 *d.* $\frac{1}{4}$ *Sterling*.

NOTE.

6 Solidi } make { 1 Groſs.

24 Groſſes } make { 1 Ducat.

EXAMPLE.

Change 512 ducats into *Sterling* money; exchange at 55*d.*$\frac{1}{2}$ *per* ducat.

D. *d.* *D.*

If 1 —— $55\frac{1}{2}$ —— 512*

2 512

512

111

2)56832(*d.*

$6\frac{1}{2}$

12)28416($1\frac{1}{2}\frac{1}{4}$

2|0)2361|8

118 *l.* 8*s.*

Ducats. *d.*

512 at $55\frac{1}{2}$ *per* Duc.

4

2048

256

64

2361|8

118 *l.* 8*s.*

---

* See Abbrevations of Multiplication.

Thirdly,

Thirdly, with SPAIN.

IN *Madrid*, *Seville*, *&c.* they keep their accompts in rials and mervadies, and exchange alſo by the piece of eight. The *par* with *London* 4*s.* 6*d.*

NOTE.

34 Mervadies } make { 1 Rial.
8 Rials } make { 1 Piece of $\frac{8}{8}$.

This exchange is alſo caſt up by the foregoing rules.

EXAMPLE.

What *Sterling* money may I draw for to *London*, if I pay in *Seville* 4798 rials, 25 mervadies; exchange at 58 *d.* *Sterling per* piece of $\frac{8}{8}$?

```
Rials.    d.     Rials.   Mer.
If 8 —— 58 —— 4798 . 25
  34               34
 ----           -------
 272             19197
                14396
                -------
                163157
                    58
               --------
               1305256
               815785
              ---------  12
         272)9463106(34790(2
             816         -------
           -----------  2|0)289|9(
              1303        -------
              1088        144 . 19 . 2¾
              ----
              2151
              1904
              ----
              .2470
              2448
              ----
              ..226
                  4
              -----
           272)904(3
               816
              -----
               .88
```

Or, back again.

Change 144*l.* 19*s.* 2*d.*$\frac{3}{4}$ *Sterling* into *Spaniſh* money; exchange at 58*d. per* piece of $\frac{8}{8}$.

```
   d.   Rials.   l.     s.   d.
If 58———8———144 . 19 . 2¾
    4           20
  ———         ————
   232         2899
                 12
              —————
              34790
                  4
              ——————
             139163
                  8
              ———————
     232)1113304(4798 Rials.
           928
          ————
           1853
           1624
          ————
          . 2290
           2088
          ————
           .2024
            1856
          ————
            .168
              34
          ————
             672
             504
              88 Rem. of the other.
          ————
        232)5800(25 Mervadies.
           464
          ————
           1160
           1160
          ————
           ...0
```

More EXAMPLES.

Change 83927 *Rials* into pounds *Sterling*; exchange at 54*d.*½ *per* piece of eight.

Anſwer, 2382*l.* 6*s.* 0*d.*½.

If I pay in *London* 5000*l.* what *Spaniſh* money may I draw my bill for to *Madrid*; exchange at 57 *d.* ⅛ *per* piece of eight?

Anſwer, 168052 *Rials*, 17 *Mervadies.*

Fourthly, with PORTUGAL.

IN *Liſbon, Oporto, &c.* they keep their accompts in Rees, and exchange on the Mill-ree; the *par* of which is about 6*s.* 8*d.*½ *Sterling.*

NOTE.

1000 Rees make 1 Mill-ree.

To make this exchange ſtill obſerve the ſame rule.

EXAMPLES.

Change 48009 rees into Sterling money; exchange at 6 *s.* 5*d.* *per* mill-ree.

```
   MR      s.   d.      Rees.
  If 1 ——6  .  5 ——48009
  1000    12               77
          ——          ———————
          77          336063
                     336063
                  ——————————— 12
        1|000)3696|693(3696)
                        4—————————
               —————————2|0)30|8
                 2|772——————————
                       15l.  8s. 0d.½
```

Or, back again,

Change 15 *l.* 8*s* 0*d.*½ *Sterling* into rees; exchange at 6*s.* 5*d. per* mill-ree.

```
  s.  d.           Rees.         l.   s.  d.
If 6 . 5 ——————— 1000 ——————— 15 . 8 . 0½
  12                            20
 ———                           ———
  77                           308
   4                            12
 ———                           ———
 308                           3696
                                  4
                               ———
                              14786
                                1000
                  —————————————— Rees.
Note, 772, Rem. of    308)14786772(48009
the other ſtating is      1232
here taken in.            ———
                          .2466
                           2464
                          —————
                             2772
                             2772
                             ———
                             ...0
```

More EXAMPLES.

What Sterling money is equal to 595677 rees; exchange at 6*s.* 7*d. per* mill-ree?

Anſwer, 196*l.* 1*s.* 6*d.*¼.

Change 59*l.* 12*s.* into rees; exchange at 6*s.* 10*d.* ½ *per* mill-ree.

Anſwer, 173381 rees.

Fifthly,

Fifthly, HOLLAND, FLANDERS, *and* GERMANY.

I Take theſe places together, becauſe their accompts are kept, and their exchange with *England* made the ſame way.

For not only in *Amſterdam*, but alſo in *Antwerp* and *Hamburgh*, they keep their accompts in pounds, ſhillings, and pence *Flemiſh*; or guilders, ſtivers, and pennicks; and exchange with us upon our pound, giving us for it, when at *par*, 33*s*. 4*d*. *Flemiſh*.

NOTE.

Their pounds, ſhillings, and pence, are divided as ours; *viz*. their pound into 20 ſhillings, and their ſhilling into 12 pence.

Their other money is thus divided.

16 Pennicks } make { 1 Stiver.
20 Stivers } make { 1 Guilder.

Note alſo,

6 Stivers } make { 1 Shilling } *Flemiſh*.
6 Guilders } make { 1 Pound } *Flemiſh*.

Sterling money is changed into *Flemiſh* by this proportion; *viz*.

As 1 *Sterling* to the given rate:
So is the given *Sterling* to the *Flemiſh* ſought.

Or,

By ſuppoſing the given Sterling, *goods*, and the given rate their *price*, caſting up their value by *practice*.

EXAM-

E X A M P L E.

If I pay in *London* 492 *l.* Sterling, what may I draw my bill for to *Amſterdam*; exchange at 34*s.* 5*d Flemiſh per* pound Sterling?

l. *s.* *d.* *d.*
If 1 —— 34 . 5 —— 492
12 413

Or by Practice, 413 1476
l. *s.* *d.* 492
492 at 34 . 5 *per l.* 1968
s $10\frac{1}{2}$ 246
$4\frac{1}{3}$ 98 . 8 12)203196(
d. $4\frac{1}{12}$ 8 . 4
$1\frac{1}{4}$ 2 . 1 2|0)1693|3(

846*l.* 13*s.* *facit.* 846*l.* 13*s.* *Fl.*

If you would have the anſwer in guilders and ſtivers, Divide the *Flemiſh* pence by 40, (the number of pence which make a guilder) and the quotient will be guilders, and if any thing remains, the half of it will be ſtivers, two *Flemiſh* pence being one ſtiver.

Or,

Multiply the *Flemiſh* pounds and ſhillings by 6, and it will produce guilders and ſtivers; for 6 guilders make 1 pound *Flemiſh*; and 6 ſtivers 1 ſhilling *Flemiſh*; and if there be any odd pence, multiply them by 8, for pennicks.

E X A M P L E.

846 *l.* 13 *s.* *the facit of the laſt ſum.*
6 *Flemiſh pence.*
4|0)20319|6(36 *Rem.*
5079 *G.* 18 *S.*
5079 *Guild.* 18 *Stivers.*

Flemiſh money (whether pounds, ſhillings, *&c.* or guilders, ſtivers, and pennicks) is exchanged into Sterling thus.

As the given rate to one pound Sterling;
So is the given *Flemiſh* to the Sterling ſought.

EXAMPLES.

Change 846 *l.* 13 *s.* *Flemiſh* into Sterling money, exchange at 34 *s.* 5 *d.* *Flemiſh per l.* Sterling.

	s.	*d. Flem.*	*l. Sterl.*	*l.*	*s. Flem.*
If	34	5 ——	1 ——	846	13
	12			20	
	413			16933	
				12	

413)203196(492 *l. Sterling.*
1652
·3799
3717
..826
826
..0

Or,

Change 5079 guilders, 18 ſtivers, into Sterling money; exchange 34 *s.* 5 *d. per l.* Sterling.

If

s. *d. Flem.* *l. Sterl.* *Guild. Stiv.*

If 34 5 —— 1 —— 5079 18

12 20

413 101598

2

413)203196(492*l. Sterl.*

1652

·3799

3717

··826

826

··0

More EXAMPLES.

Change 475 *l.* Sterling into *Flemiſh* money; exchange at 33*s. Flemiſh per l.* Sterling.

Anſwer, 783*l.* 15*s. Flemiſh.*

What *Flemiſh* money is equal to 365 *l.* 12 *s.* exchange at 36 *s.* 5*d. per l.* Sterling?

Anſwer, 665*l.* 13 *s.* 11 *d. Flemiſh.*

If I pay at *Amſterdam* 5389 *l.* 14 *s. Flemiſh*, what Sterling money ſhall I receive at *London*; exchange at 32 *s.* 8*d. Flemiſh per l.* Sterling?

Anſwer, 3299 *l.* 16 *s.* 3*d.*$\frac{3}{4}$.

Change 572 *l.* 13 *s.* Sterling into guilders, ſtivers, *&c.* exchange at 34*s.* 10*d. Flemiſh per l.* Sterling.

Anſwer, 5984 *guilders*, 3 *ſtivers*, 13 *pennicks.*

What may I draw my bill for to *London*, if I pay in *Antwerp* 7256 *guilders*; exchange at 35*s.* 2 *d.*$\frac{1}{2}$ *per l.* Sterling?

Anſwer, 686 *l.* 19 *s.* 2*d.* Sterling.

An

An Explanation of ſome Characters, ſometimes uſed for Brevity's ſake.

$+$ IS the mark of *Addition*, and ſhews the numbers it is placed between, are to be added together.

$-$ Is the mark of *Subtraction*, and ſhews the latter of the numbers it ſtands between, is to be taken from the former.

$\times$ Is the mark of *Multiplication*, and ſhews the numbers on each ſide are to be multiplied together.

$\div$ Is the mark of *Diviſion*, and placed between two numbers, ſhews the former is to be divided by the latter. Or if one or ſeveral numbers are placed, part above a line, and part below it, it ſhews the upper are to be divided by the lower, and properly ſtands for the quotient.

$=$ Is the mark of Equality, and ſhews the number or numbers preceding it, are equal to that which follows it.

$\surd$ placed before any figures, ſhews the Square root of them is meant. And

$\surd c$ before a number denotes its Cube-root.

$::$ Is the ſign of *Proportion*, and being placed betwixt the two middle terms of four proportionals, ſignifies, *ſo is*, and $:$ is generally put between the firſt and ſecond, and alſo between the third and fourth.

$\therefore$ is the mark for therefore.

To make all plain, the Characters are thus uſed:

$$2 : 4 :: 6 : 12\ \ \surd c\ 4096 = 16,\ \surd 16 = 4,\ 4 + 2 = 6,$$
$$6 - 2 = 4\ \ 4 \times 2 = 8,\ \therefore 8 \div 2 = 4.$$

And thus read:

As 2 is to 4, ſo is 6 to 12; the Cube root of 4096 is, or is equal to 16; the Square root of 16 is equal to 4; 4 added to, or more by, 2, is equal to 6; and 6 made leſs by 2 is equal to 4; again, 4 multiplied by 2 is equal to 8; therefore 8 divided by 2 is equal to 4.

CHAP. XII.

Of Arithmetical and Geometrical PROPORTION, *or* PROGRESSION.

PROGRESSION being a rule more curious than uſeful, and requiring more words for its full explication, than are conſiſtent with this *Compendium*, I ſhall only give ſome (the moſt neceſſary) propoſitions in both its parts, arithmetical and geometrical; and refer the more inquiſitive reader to Mr. *Oughtred*'s *Clavis Mathematicæ*, for his further ſatisfaction.

In all continued proportion, or compariſon of numbers, that number which is compared to another is called the *antecedent*, and the number to which it is compared is called the *conſequent*.

There are two ways of comparing numbers with one another: 1*ſt*, with reſpect only to their ſimple difference, *i. e.* how much the one, *viz.* the *antecedent*, is greater, or leſs than the other, *viz.* the *conſequent*, found by ſubtraction: thus the difference betwixt 5 and 8 is 3; which difference is called the *Arithmetical Ratio*. 2*d*. Numbers are compared with one another, when we conſider how often one number contains, or is contained in another; and this is found by diviſion, the quotient being the *geometrical ratio*. Thus, 2 is the *geometrical ratio* betwixt 8 and 16.

ARITHMETICAL PROGRESSION.

ARITHMETICAL PROGRESSION is the regular increaſe or decreaſe of any rank of numbers by the continual addition or ſubtraction of ſome equal number.

So 1, 4, 7, 10, 13, 16, are in *Arithmetical Progreſſion* to each other, equally increaſing by the continual addition of 3.

And ſo alſo are 84, 82, 80, 78, regularly decreaſing by the continual ſubtraction of 2.

Some of the moſt conſiderable properties of numbers in *Arithmetical Progreſſion* are as follow :

1. That in any rank of numbers in *Arithmetical Progreſſion*, conſiſting of three, or any odd number of terms, the double of the middle term will be equal to the ſum of the two extremes, or of any two mean, equally diſtant from the ſaid middle term.

1, 2, 3, 4, (5), 6, 7, 8, 9.

Thus in the rank above 5, the middle term doubled is 10, =to 1+9, the two extremes, or 2+8, or 3+7, or 4+6, the ſeveral correſpondent means.

2. That in any rank of numbers in *Arithmetical Progreſſion* conſiſting of four, or any even number of terms, the ſum of the two extremes will be equal to the ſum of the two middle numbers, or of any two means equally diſtant from the ſaid extremes.

2, 4, 6, 8, 10, 12, 14, 16.

Thus in the rank above 2+16, the two extremes make 18,=8+10, the two middle numbers, or 6+12, or 4+14, the other correſpondent means.

In PROGRESSION *five things are to be noted.*

VIZ.

I. The firſt Term.
II. The laſt Term.
III. The number of Terms.
IV. The equal Difference.
V. The ſum of all the Terms.

Any three of which being given, the other two may be found; as Mr. *Oughtred* in his *Clavis Mathematicæ*, *Chap*, 19. *Prop*. the 6th, exemplifies in 20 propoſitions; ſome of the moſt uſeful of which are as follow:

PROPOSITION. I.

The laſt term, number of terms, and equal difference being given, to find the firſt term:

Or, the ſecond, third, and fourth given, to find the firſt.

RULE.

Multiply the fourth by the third, made leſs by one, and ſubtract the product from the ſecond, the remainder is the anſwer.

EXAMPLE.

A man takes out of his pocket, at 6 ſeveral times, 6 ſeveral numbers of ſhillings, every one exceeding the former by 6; the laſt was 42, what was the firſt? *Anſwer*, 12*s*.

$6 \times 5 = 30$, $42 - 30 = 12$ *Anſwer*.

PROPOSITION II.

The firſt, third, and fourth given, to find the ſecond.

RULE.

From the product of the fourth, multiplied by the third, ſubtract the fourth; the remainder added to the firſt, gives the ſecond.

EXAMPLE.

What's the laſt number of an *Arithmetical Progreſſion* beginning at 4, and continuing, by the increaſe of 12 to 18 places? *Anſwer*, 208.

12×18=216, 216−12=204, 204+4=208 *Anſwer.*

PROPOSITION III.

The firſt, ſecond, and fourth given, to find the third.

RULE.

Subtract the firſt from the ſecond, and divide the remainder by the fourth; the quotient, with one added, gives the third.

EXAMPLE.

A traveller went 12 miles the firſt day, and increaſed every day's journey by 4 miles, till at laſt he went 64 miles in one day; how many days did he travel? *Anſwer*, 14 days.

64−12=52, 52÷4=13, 13+1=14 *Anſwer.*

PROPOSITION IV.

The firſt, ſecond, and third given, to find the fourth.

RULE.

RULE.

Subtract the firſt from the ſecond, and divide the remainder by the third, made leſs by one, the quotient gives the fourth.

EXAMPLE.

The ages of ſeven children increaſe by *Arithmetical Progreſſion*; the youngeſt is two years old, the eldeſt 32: what's the common difference of their ages? *Anſwer*, 5 years.

$32-2=30$, $30\div 7-1=5$ *Anſwer.*

PROPOSITION V.

The firſt, ſecond, and third given, to find the fifth.

RULE.

Half the ſum of the firſt and ſecond, multiplied by the third, produces the fifth.

EXAMPLE.

12 perſons give their charity to a poor man in *Arithmetical Progreſſion*; the firſt gave 2 pence, the laſt 2 ſhillings, or 24 pence: how much did the poor man get? *Anſwer*, 156 pence, or 13 ſhillings.

$2+24=26$, $26\div 2=13$, $13\times 12=156$ *Anſwer.*

GEOMETRICAL PROGRESSION.

GEOMETRICAL PROGRESSION is the increaſing or decreaſing of any rank of numbers by ſome equal *ratio*, that is, by the continual multiplication or diviſion of ſome equal number.

Thus,

Thus 2, 4, 8, 16, 32, increaſe in *Geometrical Progreſſion*, by a double *ratio*, or continual multiplication by 2; and 405, 135, 45, 15, 5, decreaſe after the ſame manner by a continual diviſion by 3, or by a triple *ratio*.

Some of the moſt conſiderable properties of numbers in *Geometrical Proportion* are as follow:

1. That in any rank of numbers in *Geometrical Progreſſion*, conſiſting of three, or any odd number of terms, the product of the middle term, multiplied by itſelf, will be equal to the product of the two extremes, or of any two means equally diſtant from the ſaid middle term.

1, 2, 4, (8), 16, 32, 64.

Thus, in the rank above 8, the middle term ſquared *i. e.* multiplied by itſelf, is 64=1×64, the two extremes, or 2×32, or 4×16, the correſpondent means.

2. That in any rank of numbers in *Geometrical Progreſſion*, conſiſting of four, or any even number of terms, the product of the two extremes will be equal to the product of the two middle numbers, or of any two means equally diſtant from the ſaid extremes.

2, 4, 8, 16, 32, 64.

Thus in the rank above 2×64, the two extremes produce 128=8×16, the two middle numbers, or to 4×32, the other correſpondent means.

In Geometrical Progreſſion *the ſame five things are to be noted as in* Arithmetical Progreſſion:

VIZ.

I. The firſt Term.
II. The laſt Term.
III. The number of Terms.
IV. The *ratio*, or equal difference.
V. The ſum of all the Terms.

Note, Figures are ſometimes ſet over a rank of geometrical proportionals, as here-under, and are called *Indices,* or *Exponents* ; ſerving to ſhew the diſtance of any term from unity, or from the firſt term of the ſeries.

$$\text{Thus}\ \begin{array}{ccccccc}0&1&2&3&4&5&6\\1\,.&2\,.&4\,.&8\,.&16\,.&32\,.&64\end{array}$$

PROPOSITION I.

To find any remote term of a *Geometrical Progreſſion,* proceeding from unity, whoſe *ratio,* or common difference is known, without producing all the intermediate terms.

RULE.

Find ſome few of the leading terms, and place over them their exponents ; then multiplying the laſt found term by itſelf, it will produce a term double thereto ; which again multiplied by itſelf, will produce another double to the laſt : Thus proceed till either you produce the term ſought, or one a little ſhort of it ; which may be completed by multiplying it again by that term, which ſtands under ſuch exponent as would make up the number.

Or, in other words, obſerve what figures of the exponents found added together, would give the exponent of the term wanted, and the numbers ſtanding under the ſaid exponents, multiplied into each other, will produce the term required.

From whence may be obſerved the coherence or agreement betwixt numbers in arithmetical and geometrical proportion.

EXAMPLE.

One agrees for 14 oranges, to pay only the price of the laſt, at a farthing for the firſt, an half penny the ſecond, *&c.* ſtill doubling the price for the next, what muſt he give ?

Here, ſetting down only the five firſt terms, with their exponents, the fourteenth may thus be found, 5+5+4=14. So the term under thoſe exponents being multiplied into each other, give the term anſwering. But as the exponent 1 ſtands always over the ſecond term, ſo the exponent 13 anſwers to the 14th term; therefore add only 3 to the two 5's; and ſo always that exponent muſt be taken, which is one ſhort of the term required; as for 20, 19; for 30, 29, *&c.* which muſt be carefully noted.

See the work of the foregoing example.

0 . 1 . 2 . 3 . 4 . 5
1 . 2 . 4 . 8 . 16 . 32

Exponents.	*Terms.*
5	32 *the 5th multiplied by*
5	32 *itſelf.*
3	64
13	96
	gives 1024 *the 10th, which*
	multiplied by —8 *the 3d Term,*
	gives 8192 *the 14th.*

And ſo many farthings, or 8 *l.* 10 *s.* 8 *d.* muſt he give at the rate agreed.

PROPOSITION II.

To find any remote term of a *Geometrical Progreſſion*, not proceeding from unity, whoſe *ratio* and firſt term is given, without producing all the intermediate terms.

RULE.

RULE.

Set down, as before, ſome few of the leading terms with their exponents, and multiply alſo (as in the firſt propoſition) the terms under ſuch exponents, as added, would make the exponent ſhort by one of the terms required; only remember, that every product muſt here be divided by the firſt term.

EXAMPLE.

A ſum of money is thus to be divided among 9 perſons, the 1ſt to have 50*l.* the 2d 150*l.* and ſo on in that proportion, one three times more than the other, to the laſt; what will the laſt have?

Setting down the five firſt terms with their exponents, thus,

```
 0    1    2     3     4
50  150  450  1350  4050
```

Multiply the laſt term by itſelf, and divide the product by 50, the firſt term, the quotient will be 328050 the 9th term required, anſwering to the 8th exponent.

Or ſetting down but the three firſt terms, multiply the laſt by itſelf, and divide the product by the 1ſt term, it quotes the 5th; which again multiplied by itſelf, and divided by the firſt, will quote the 9th, as before. See the work.

```
 0   1    2         450             4050
50  150  450        450             4050
                 ---------        ---------
                  22500            202500
                 1800             162000
                 ---------        ------------
             5|0)2025|0(     5|0)1640250|0(
                 ---------        ---------
                  4050             328050
```

Anſwer, 328050*l.* for the 9th perſon.

PROPOSITION III.

Having the firſt term, the common exceſs, and the number of places given, to find the total ſum of the whole progreſſion.

RULE.

Having found the laſt term by the foregoing rules, ſubtract from it the firſt, and divide the remainder by the common exceſs, made leſs by one; the quotient is the ſum of all, except the laſt: to which that being alſo added, gives the total required.

Note, If the *ratio* be double, that is, if the common exceſs be 2, the difference between the leaſt and the greateſt term, added to the greateſt, gives the total ſum.

If the *ratio* be triple, $\frac{1}{2}$ the difference muſt be added; if quadruple, $\frac{1}{3}$, &c.

EXAMPLES.

1*ſt*. A coffee-man, upon the ſigning of the laſt peace, for 50 guineas down, agreed to pay the firſt day one coffee-berry, the ſecond two, the third four, and ſo on, to double the quantity every day, till the ſame was proclaimed; what number of berries would it amount to, ſuppoſing the time 60 days: and what would their value be, ſuppoſing 100 berries to the pound, and the pound to be ſold at 5 *s*?

See the work.

Days.

1—— 1 *Days.*
2—— 2
3—— 4
4—— 8
5——16

16 *Berries.*
16
96
16
256
2
512 *the* 10*th Day.*
16
3072
512
8192
2
16384 *the* 15*th Day.*
16348
65536
131072
49152
98304
16384
268435456
2
536870912 *the* 30*th Day.*
536870912
1073741824
536870912
4831838208
37580963840
4294967296
3221225472
1610612736
2684354560
288230376151711744
2
576460752303423488 *the* 60*th Day.*
1|000)1152921504606846|975 *Tot. number of berries.*
5 *s.* ¼ 288230376151711*l.* 10*s. their val.* at 5*s. per* ℔.

Ex. 2. A Grazier offers 40 oxen for a farthing a head, and treble it throughout; to what ſum will it amount?

See the work.

Oxen. *Farthings.*

1 —— 1
2 —— 3
3 —— 9
4 —— 27
5 —— 81
81

81
648

6561
3

19683 10*th Ox.*
19683

59049
157464
118098
177147
19683

387420489
3

1162261467 *the* 20*th Ox.*

1162261467
1162261467

8135830269
6973568802
4649045868
1162261467
6973568802
2324522934
2324522934
6973568802
1162261467
1162261467

1350851717672992089
3

4052555153018976267 40*th Ox.*

6078832729528464400 *Tot. Far.*
Or,
6332117426592150*l.* 8*s.* 4*d.*

CHAP.

CHAP. VIII.

Of VULGAR FRACTIONS.

A FRACTION is a part or parts of an unit, and written with two figures, with a line drawn between them, as $\frac{3}{4}$, $\frac{2}{5}$, $\frac{6}{8}$, each of which really ſtands for the quotient of a diviſion made of the upper figure divided by the lower; ſo that if the upper figure can be multiplied into any number which the unit is equal to in known parts, and that product divided by the lower figure, the quotient will give the number of thoſe parts equal to the fraction.

As $1l. = 20s.\ \frac{3}{4}l. = \frac{60}{4}s. = 15s.$

Or,

$1s. = 12d.\ \frac{3}{4}s. = \frac{36}{4}d. = 9d.$

The figure under the line is called the *denominator*, becauſe it gives name to the fraction, and alſo ſhews into what parts the unit is broken; and that above, the *numerator*, becauſe it tells how many of ſuch parts are meant by the fraction. Thus the fraction $\frac{3}{4}$ ſhews that the unit is divided into 4 parts, and that 3 of thoſe parts are thereby expreſs'd.

1. Therefore, if the *numerator* and *denominator* of a fraction ſhould be equal, as $\frac{4}{4}$, $\frac{6}{6}$, the value of ſuch fraction would be exactly an unit or integer; for, by the definition above, the *denominator* ſhews into how many parts the unit is broke: and the *numerator* expreſſes how many of thoſe parts, are meant by the fraction: So that in the fraction $\frac{4}{4}$, the *denominator* declaring the unit to be broke into 4 parts, and the *numerator* expreſſing 4, that is, all of thoſe parts, it is plain the ſaid fraction is equal to an unit or whole number; becauſe the ſum of all the parts muſt be equal to the whole. From which obſervation it is alſo plain, that ſo often as the *denominator* is contain'd in the *numerator*, ſo many whole numbers or units are

contain'd in ſuch a fraction. And this may ſerve as a reaſon for the operation of the 1ſt, 2d and 3d ſorts of Reduction.

2. As all proper fractions may be ſuppoſed to ariſe from the remainders of diviſions, when the diviſor can no longer meaſure the dividend; ſo every fraction may be look'd upon as the two given terms of a diviſion; the *numerator* as the dividend, and the *denominator* as the diviſor, according to the latter part of the character, *page* 128, to ſignify diviſion; from whence it appears that if the *numerator* and *denominator* of a fraction be either multiplied or divided both by the ſame number, the products or quotients will ſtill remain in the ſame proportion, and the new fraction ſo ariſing be of the ſame value with that given. Thus the *numerator* and *denominator* of the fraction $\frac{2}{4}$, multiplied by 2, will produce $\frac{4}{8}$; or divided by 2, will quote $\frac{1}{2}$; all which fractions are of the ſame value; 4 bearing the ſame proportion to 8, and 1 to 2, as 2 does to 4. And from hence may appear the reaſon of the 5th and 6th ſorts of Reduction.

Of Vulgar Fractions *there are four ſorts*, viz.

1ſt. A *proper*; whoſe *numerator* is leſs than its *denominator*, as $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{8}$, *&c.*

2d, An *improper*; whoſe *numerator* is equal to, or greater than its *denominator*, as $\frac{5}{3}$, $\frac{6}{4}$, *&c.*

3d. A *compound*, or *fraction of a fraction*; known by the word *of*, as $\frac{3}{4}$ *of* $\frac{5}{6}$, *&c.*

4th. That which is join'd with a whole number, as $5\frac{1}{2}$, $3\frac{2}{3}$, *&c.* call'd a *mix'd number*.

Before fractions can be either added or ſubtracted, they muſt be reduced into one denomination, that is, each to have the ſame *denominator*; and therefore, before we proceed to thoſe rules, we muſt learn Reduction.

REDUCTION *of* FRACTIONS.

IN *Reduction of Fractions* there are ſeven varieties.

Firſt, To reduce a whole number into an *improper Fraction*; which is done by placing 1 for its *denominator*.

EXAMPLES.

Reduce 4 to a Fraction, *facit* $\frac{4}{1}$.
or 18,————————*facit* $\frac{18}{1}$.

But if you would aſſign it any other *denominator*, multiply the whole number by the *denominator* aſſign'd, and place the product for a *numerator* over the aſſign'd *denominator*.

EXAMPLES.

Reduce 12 to a fraction, whoſe *denominator* let be 8.
$\begin{array}{r}\underline{8}\\96\end{array}$ *facit* $\frac{96}{8}$.

Or if 6 were to be made a fraction, and its *denominator* to be 7, it would become $\frac{42}{7}$.

For the reaſon of this rule conſider the foregoing remarks.

Second, To reduce a mixed number into an improper one.

Multiply the integral parts of the number by the *denominator* of the fraction given, and take in the *numerator*; the product place for a new *numerator* over the *denominator* given.

EXAM-

EXAMPLES.

Reduce $5\frac{1}{3}$ to an improper fraction, *facit* $\frac{16}{3}$.

$$\begin{array}{c} 3 \\ \hline 16 \\ \hline 3 \end{array}$$

So $12\frac{1}{2}$ reduc'd to an improper fraction, will be $\frac{25}{3}$.

The reaſon of this rule is the ſame as the foregoing, there being no difference in the operation, but the taking in the given *numerator*.

Third, To reduce an improper fraction into its equivalent whole or mix'd number.

RULE.

Divide the *numerator* by the *denominator*, the quotient gives the whole number contain'd: But if any thing remains, (as in the ſecond example) it muſt be plac'd as a new *numerator* over the given *denominator*.

EXAMPLES.

Reduce $\frac{32}{8}$ to its equivalent whole number.

$$\begin{array}{c} 8)32(4 \textit{ facit.} \\ \hline 0 \end{array}$$

Reduce $\frac{45}{6}$ to its equivalent whole or mix'd number.

$$\begin{array}{c} 6)45(7\frac{3}{6} \textit{ facit.} \\ 42 \\ \hline 3 \end{array}$$

This being only the reverſe of the foregoing rule, the ſame reaſon ſtill holds.

Fourth, To reduce a compound fraction to a ſingle one, *i. e.* a fraction of a fraction, to a fraction of an unit, which is the multiplication of them into one another.

RULE.

Multiply all the *numerators* together for a *numerator*, and all the *denominators* for a *denominator*, the fraction thus form'd is the ptoduct.

EXAMPLES.

Reduce $\frac{2}{3}$ of $\frac{1}{4}$ to a ſingle fraction, *facit* $\frac{2}{12}$.

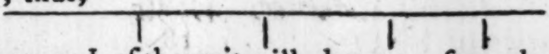
Reduce $\frac{4}{5}$ of $\frac{6}{8}$ of $\frac{7}{9}$ to a ſingle fraction, *facit* $\frac{168}{360}$.

See the work——

$$\begin{array}{cc} 4 & 5 \\ 6 & 8 \\ \hline 24 & 40 \\ 7 & 9 \\ \hline 168 & 360 \end{array}$$

So alſo $\frac{3}{8}$ of $\frac{9}{10}$ is $\frac{27}{80}$.

The reaſon of this operation will beſt appear by repreſenting the unit by a line, which, according to the firſt example, muſt be ſuppoſed to be divided into 4 parts, and each of thoſe parts again into 3 ſmaller parts; thus,



Then, as $\frac{1}{4}$ of the unit will denote 1 ſpace between the larger diviſions, ſo $\frac{2}{3}$ of that fourth muſt ſignify only 2 ſpaces of the leſſer diviſions; conſequently, if I would expreſs what part of the whole line $\frac{2}{3}$ of $\frac{1}{4}$ is, it will plainly appear to be $\frac{2}{12}$: or, had the compound fraction been $\frac{2}{3}$ of $\frac{3}{4}$, the ſingle fraction equal to it had been $\frac{6}{12}=\frac{1}{2}$. This ſhews that multiplying a fraction or any quantity by a proper fraction, decreaſes it in the ſame proportion as ſuch proper fraction is leſs than an unit.

Fifth, To reduce a fraction into its loweſt terms.

RULE.

Divide the *numerator* and *denominator* by any figure, ſo that nothing may remain, and the quotients will be a new fraction of the ſame value with that given.

The rule generally given for finding the greateſt common meaſure, or number, to divide your given fraction by, is this:

Divide the *denominator* by the *numerator*, and the *numerator* by the *remainder*, if any: ſo continuing to make the laſt diviſor the dividend, and the remainder the diviſor, till nothing remains, the laſt diviſor will be the greateſt common meaſure, or number, by which you can divide your fraction.

EXAMPLE.

What is the greateſt common meaſure by which $\frac{208}{684}$ can be divided? & p. 31.

208)684(3
624

60)208(3
180

28)60(2
56

Anſwer, 4)28(7
28

0

$\frac{208}{684}$ then, being divided by 4, gives $\frac{52}{171}$, which are its loweſt terms.

But this way of finding the common meaſure is too tedious, often making more work than it ſaves. Obſerve therefore theſe more practical directions.

If you cannot at once diſcover the greateſt number you may divide by, if both your *numerator* and *denominator* are even numbers, you may always halve them, and often that way reduce your fraction to advantage.

Thus the fraction given in the foregoing example divided twice by 2, gives, as before, $\frac{52}{171}$.

$$2\ \ \frac{208}{684}\Big|\frac{104}{342}\Big|\frac{52}{171},$$

and $\frac{192}{576}$ may be reduced, by continual halving, to $\frac{3}{9}$.

$$2\ \ \frac{192}{576}\Big|\frac{96}{288}\Big|\frac{48}{144}\Big|\frac{24}{72}\Big|\frac{12}{36}\Big|\frac{6}{18}\Big|\frac{3}{9}.$$

Alſo, when both your *numerator* and *denominator* have cyphers on the right hand, you may abbreviate the fraction, by ſtriking off an equal number from both.

Thus $\frac{45}{66}\Big|\frac{00}{00}$, or $\frac{825}{990}$.

Or, if the right hand figures of your *numerator* and *denominator* are both fives, or one a five, and the other a cypher, as the remainder of the laſt example) you may always divide them by 5.

Thus $\frac{825}{940}$ divided by 5.
is $\frac{165}{192}$, equal to $\frac{825}{960}$.
So alſo is $\frac{125}{365}$ reduced to $\frac{25}{73}$.

Thus in the Rule of Three, we abbreviate our ſtatings by the ſame rule, and on the ſame reaſon ; for the ſecond and third numbers multiply'd together, are as a *numerator* to the firſt number, their *denominator*.

For the reaſon of this rule, ſee the ſecond remark.

Sixth, to reduce ſeveral fractions given with different denominators to have a common denominator.

RULE.

Multiply each of the given *numerators* into all the given *denominators*, except its own ; and the ſeveral products will be ſo many new *numerators*, whoſe common *denominator* muſt be the product of all the given *denominators* multiply'd together.

EXAMPLES.

Reduce $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{6}{10}$ to have a common denominator.

To do which,

Firſt, The *numerator* 3 muſt be multiply'd by the *denominators* 8 and 10, which will produce 240.

Secondly, the *numerator* 5, multiply'd by the *denominators* 4 and 10, will produce 200.

Thirdly, The *numerator* 6, multiply'd by the *denominators* 4 and 8, will produce 192.

New numerators.

Then the three *denominators* 4, 8, 10, multiply'd together, will give 320 for a common *denominator*.

See the work.

1ſt Numb.	*2d Numb.*	*3d Numb.*	*Denomin.*
3	5	6	4
8	4	4	8
24	20	24	32
10	10	8	10
240	200	192	320

$\frac{240}{320}$ $\frac{200}{320}$ $\frac{192}{320}$

So $\frac{5}{10}$, $\frac{3}{8}$, $\frac{4}{12}$, and $\frac{9}{15}$, reduced to have a common Denominator will be $\frac{7200}{14400}$, $\frac{5400}{14400}$, $\frac{4800}{14400}$, $\frac{8640}{14400}$.

See the work.

1ſt Num.	*2d Num.*	*3d Num.*	*4th Num.*	*Deno.*
5	3	4	9	10
8	10	10	10	8
40	30	40	90	80
12	12	8	8	12
480	360	320	720	960
15	15	15	12	15
7200	5400	4800	8640	14420

The foregoing is the common Rule for reducing ſeveral fractions with different denominators, to have a common denominator, but as, by abbreviation fractions are often reduced into much lower terms, than they are at firſt given; ſo here, a common denominator may ſometimes be found much ſmaller than that ariſing by the preceding method in the following manner.

RULE.

Having firſt, by the fifth ſort of Reduction, reduc'd all your given fractions to their loweſt terms, cancel or ſtrike out all ſuch of the *denominators* as are *aliquot* parts of others; then drawing a line under all the *denominators*, if two or more can be divided by any number, ſo that nothing may remain, divide by ſuch number; and place the quotients under the line, and with them, in their proper places, all ſuch of the *denominators* as would not admit of ſuch diviſion. If any of the ſaid quotients or *denominators* ſo brought down, are capable of further abbreviation, draw another line, and proceed as before, ſo continuing as low as poſſible; then multiply continually, one into another, the ſeveral diviſors, and the quotients ariſing, (with the remaining denominators if any) under the laſt line, and their product will be the leaſt common *denominator*.

The new *numerators* to which may be found, by dividing the ſaid common *denominator* by each given *denominator*, and multiplying the quotients by their reſpective *numerators*.

EXAMPLE.

Reduce $\frac{3}{4}\ \frac{5}{8}\ \frac{6}{10}$ to have the leaſt common denominator 2 ——

Anſwer, $\frac{30}{40}\ \overset{4}{\frac{25}{40}}\ \overset{5}{\frac{24}{40}}$.

The common *denominator* 40 is thus produced.

Firſt, 'The *denominator* 4 being an *aliquot* part of 8, is cancelled, then the *denominators* 8 and 10 are divided by 2. *Laſtly*, the diviſor 2×4×5=40.

The reaſon of this ſixth ſort of reduction is evident from the 2d of the preceding remarks; for as both the *numerator* and *denominator* of each given fraction are equally multiply'd by all the other *denominators*, conſequently the new fractions thence ariſing, muſt be equal to the fractions given.

Seventh, To value a fraction, or reduce it to the known parts of an integer.

RULE.

Multiply the *numerator* by the next inferior denomination, and divide the product by the *denominator*; the quotient ſhews the parts ſought, and the remainder becomes a new *numerator* to the given *denominator*; which muſt ſtill be valued by the ſame rule, proceeding till you have brought it into the leaſt known parts of the integer.

e. h. 25

EXAM-

EXAMPLES.

1*ſt.* What's the value of $\frac{3}{5}$ of a pound ſterling?

3
20
——
5)60(
12 *Shillings.*

2*d.* What's the value of $\frac{13}{49}$ of a pound ſterling?

13
20
—— *s. d. qrs.*
49)260(5 . 3 . 2$\frac{14}{49}$
——
.15
12
——
49)180(3
——
.33
4
——
49)132(2
——
.34

3*d.* What hundreds, quarters, and pounds, are contained in $\frac{53}{94}$ of a ton?

53
20
—— *C qr.* ℔.
94)1060(11 . 1 . 2$\frac{92}{94}$
——
.120
——
.26
4
——
94)104(1
——
.10
28
——
94)280(2
——
.92

There will be no difficulty in accounting for this rule, if we conſider but the particular working of any one example: Thus in the firſt, $\frac{3}{5}$ of a pound ſterling are given to be valued: Now as 20*s.* make a pound, ſo conſequently any part of a pound muſt be 20 times as great a part of a ſhilling: Therefore $\frac{3}{5}$ of a pound make $\frac{60}{5}$ of a ſhilling; which being an improper fraction its *numerator* is divided by its *denominator,* to find the units or whole numbers (which in this caſe muſt be ſhillings) contained in it; according to the directions of the 3d ſort of Reduction.

By this rule are remainders of ſtatings in the Rule of Three, &c. valued. See page 25.

ADDITION *of* FRACTIONS.

WHEN the given fractions are parts of one common denominator, all you have to do is, to add the numerators together, and place the ſum for a new numerator over the given common denominator; thus $\frac{1}{6}$, $\frac{4}{6}$ and $\frac{3}{6}$ added together, make $\frac{8}{6}$ equal to $1\frac{2}{6}$.

But when amongſt the given fractions there are either compound ones or ſingle with different denominators, they muſt be prepared by reduction, before they can be added.

EXAMPLES.

1ſt. What's the ſum of $\frac{3}{6}$, $\frac{2}{9}$, and $\frac{4}{7}$?

According to the ſixth ſort of reduction the given fractions being reduced to a common denominator they will be $\frac{189}{504}$, $\frac{112}{504}$. $\frac{288}{504}$, the numerators of which being added, the ſum is $\frac{189}{504}$ equal to $1\ \frac{85}{504}$.

2d. Add $\frac{3}{4}$ of $\frac{3}{8}$, and $\frac{9}{10}$ of $\frac{6}{7}$ and $\frac{5}{8}$ together.

Reduce the compound fractions to ſingle ones, by the fourth rule of reduction; and inſtead of $\frac{3}{4}$ of $\frac{3}{8}$, and $\frac{9}{10}$ of $\frac{6}{7}$, you will have $\frac{9}{32}$ and $\frac{54}{70}$, to add to your $\frac{5}{8}$; all which ſingle fractions being reduced, firſt into their loweſt terms, and then to a common denominator, they will be $\frac{315}{1120}$, $\frac{864}{1120}$ and $\frac{700}{1120}$; which added, make $\frac{1879}{1120}$, equal to $1\ \frac{759}{1120}$.

3*d.* Add 4 $\frac{1}{2}$ and 3 $\frac{1}{6}$ and $\frac{2}{7}$ together.

When mix'd numbers are given to be added, reduce only their fractional parts to one denominator, and add their ſum to the total of the whole numbers.

$$\begin{array}{rcr}
\frac{42}{84}\ \frac{14}{84}\ \frac{24}{84}\ 4 & \text{———} & 42\\
3 & \text{———} & 14\\
 & & 24\\
\hline
 & & 80\\
\textit{facit}\ 7 & \text{———} & \\
 & & 84
\end{array}$$

More EXAMPLES.

Add 8 *l.* $\frac{1}{2}$ and 3 *l.* $\frac{9}{10}$ and $\frac{17}{25}$ together.

Facit 13 *l.* $\frac{4}{50}$.

What's the ſum of $\frac{3}{4}$ of $\frac{7}{8}$, and 4 *l.* $\frac{1}{2}$ and $\frac{10}{20}$?

Anſwer, 5 *l.* $\frac{379}{480}$.

SUBTRACTION *of* FRACTIONS.

S*ubtraction of Fractions* is alſo nothing (after the given fractions are prepared by reduction) but taking on *numerator* from the other.

EXAMPLES.

1ſt. *From* $\frac{3}{4}$ *take* $\frac{4}{9}$ 27
16
—
prepared $\frac{27}{36}$ $\frac{16}{36}$ $\frac{11}{36}$ *Remainder.*

2d. *From* $5\frac{2}{3}$ *take* $\frac{9}{10}$.

Firſt, The mix'd number $5\frac{2}{3}$ being reduced to the improper fraction $\frac{17}{3}$, by the ſecond rule of reduction proceed as before.

From $\frac{17}{3}$, *take* $\frac{9}{10}$, or *prepared.*
From $\frac{170}{30}$ *take* $\frac{27}{30}$ *Remains* $\frac{143}{30}$ *or* $\frac{23}{30}$.

Or without reducing the mixed nnmber to an improper fraction, make only the fractional parts of one denomination, and you will then have $\frac{27}{30}$ to take from $\frac{20}{30}$; which becauſe you cannot do, you muſt borrow an unit, *i. e.* $\frac{30}{30}$; then, $\frac{27}{30}$ from $\frac{30}{30}$, there will remain $\frac{3}{30}$; to which adding the $\frac{20}{30}$, the remainder will then be $\frac{23}{30}$; and the unit borrowed being deducted from the whole number 5, there will remain in all $4\frac{23}{30}$, as before.

More EXAMPLES.

From 5*l.* $\frac{7}{8}$ *take* $\frac{9}{10}$ of $\frac{5}{6}$ *Rem.* 5*l* $\frac{1}{8}$
From $\frac{19}{20}$ *take* $\frac{1}{7}$ of $\frac{2}{3}$ *Rem.* $\frac{359}{420}$.

MULTIPLICATION *of* FRACTIONS.

IF the numbers given are whole, or mixed, they muſt firſt be brought into improper fractions; or if they are compound fractions, they muſt be reduced to ſingle ones, or taken as part of the fractions to be multiplied together; but if the numbers given are either proper or improper ſingle fractions, you have only to multiply the two given *numerators* together for a new *numerator* and the given *denominators* for a *denominator*. The reaſon of which is clear, from the explanation of the fourth ſort of reduction, which is this rule itſelf.

EXAMPLES.

1*ſt*. Multiply $\frac{4}{8}$ by $\frac{3}{6}$, *facit* $\frac{12}{48}$, or by abbreviation $\frac{1}{4}$.

2*d*. Multiply $\frac{9}{6}$ by $\frac{4}{7}$, *facit* $\frac{36}{42}$, or $\frac{6}{7}$.

3*d*. Multiply 4 by $\frac{3}{8}$. 4 by the firſt ſort of reduction, reduced to a fraction, is $\frac{4}{1}$; which multiplied by $\frac{3}{8}$, makes $\frac{12}{8}$, or 1 $\frac{1}{2}$.

4*th*. Multiply 7$\frac{1}{2}$ by 8$\frac{2}{3}$. Theſe mixed numbers reduced by the 2d rule of reduction to improper fractions, make $\frac{15}{2}$ and $\frac{26}{8}$; which, multiplied together, make $\frac{390}{6}$, or 65.

It may here be required to ſhew how mix'd numbers may be multiplied together without reducing them to improper fractions, which in many caſes may ſhorten the work.

Suppoſe it, as in the laſt example, required to multiply 7 $\frac{1}{2}$ by 8 $\frac{2}{3}$.

Firſt, 8 times $\frac{1}{2}$ or ($\frac{1}{2}$ of 8 is 4, without any fraction remaining, which carried to (8 times 7) 56 makes 60 to be ſet down as in multiplication of integers; now to multiply by $\frac{2}{3}$, we may firſt take $\frac{1}{3}$ of 7$\frac{1}{2}$, thus, $\frac{1}{3}$ of 7 is 2, and 1 remains, which makes $\frac{1}{2}$ up $\frac{3}{2}$, and $\frac{1}{3}$ of $\frac{3}{2}$ is $\frac{1}{2}$, ſo 2$\frac{1}{2}$ twice ſet down, and added to the 60, make up the product 65, as before.

$$\begin{array}{r} 7\,\frac{1}{2} \\ 8\,\frac{2}{3} \\ \hline 60 \\ 2\,\frac{1}{2} \\ 2\,\frac{1}{2} \\ \hline 65 \end{array}$$

Again, ſuppoſe it required to multiply $3\frac{3}{4}$ by $3\frac{3}{4}$.

Firſt, 3 times $\frac{3}{4}$ is $(\frac{9}{4})$ $2\frac{1}{4}$. I ſet down the $\frac{1}{4}$, and carry the 2 to (3 times 3) 9, which makes 11 to be ſet down as an integer; now $\frac{3}{4}$ of $3\frac{3}{4}$ may be taken as in the rule of Practice, firſt the half of it, and then the half of that half, as the $\frac{1}{2}$ of 3 is 1, and one remains, that makes the $\frac{3}{4}$ up $\frac{7}{4}$, the half of $\frac{7}{4}$ is $\frac{7}{8}$; now half of $1\frac{7}{8}$ is $\frac{15}{16}$. Theſe fractions are eaſily reduced, and added up with the integers.

$$\begin{array}{r}
3\frac{3}{4} \\
3\frac{3}{4} \\
\hline
11\frac{1}{4} \\
1\frac{7}{8} \\
\frac{15}{16} \\
\hline
14\frac{1}{16}
\end{array}$$

Let it be required to multiply $892\frac{3}{11}$ by $78\frac{7}{8}$.

Firſt, 8 times 892 is 7136, and 70 times 892 is 62440; then taking the $\frac{7}{8}$ of 892 by ſeveral parts as in Practice, thus, half 892 is 446, half of 446 is 223, and half of 223 is $111\frac{1}{2}$; now for the $\frac{3}{11}$ of $78\frac{7}{8}$, firſt 3 times $78\frac{7}{8}$ is $236\frac{5}{8}$, which divided by 11, quotes $21\frac{45}{88}$, and theſe added is the product, as ſhewn at the ſide.

$$\begin{array}{r}
892\frac{3}{11} \\
78\frac{7}{8} \\
\hline
7136 \\
6244 \\
446 \\
223 \\
111\frac{1}{2} \\
21\frac{45}{88} \\
\hline
70378\frac{1}{88}
\end{array}$$

More *EXAMPLES.*

Multiply $9\frac{1}{2}$ by $\frac{2}{6}$, *facit* $3\frac{1}{6}$.
Multiply $\frac{7}{8}$ by 8, the product is 7.

This laſt example helps to clear up the reaſon given for the rule in Diviſion, by ſhewing that multiplying a fraction by its denominator takes it away, and makes the numerator a whole number.

DIVISION *of* FRACTIONS.

WHEN you have made the ſame preparation of your number as directed in Multiplication, multiply the numerator of your dividend by the denominator of your diviſor, for a numerator; and the numerator of your diviſor by the denominator of your dividend, for a denominator; the fraction thus formed, is the quotient.

EXAMPLES.

1*ſt*. Divide $\frac{7}{9}$ by $\frac{1}{8}$, *facit* $\frac{56}{27}$, or 2 $\frac{2}{27}$.

2*d*. Divide $\frac{7}{8}$ by $\frac{1}{3}$, *facit* $\frac{21}{8}$, or 2 $\frac{5}{8}$.

3*d*. Divide 7 by $\frac{3}{4}$, prepared by the firſt rule of Reduction, it will be $\frac{7}{1}$ by $\frac{3}{4}$; *facit* $\frac{28}{3}$, or 9 $\frac{1}{3}$.

4*th*. Divide 4$\frac{1}{2}$ by 2$\frac{3}{8}$, by the 2d rule, it is $\frac{2}{2}$ by $\frac{19}{8}$, which divided gives $\frac{72}{38}$, or 1 $\frac{34}{38}$.

5*th*. Divide $\frac{2}{3}$ of $\frac{1}{9}$ by $\frac{4}{8}$ of $\frac{7}{8}$, reduced by the 4th Rule $\frac{2}{27}$ by $\frac{28}{64}$, *facit* $\frac{128}{756}$.

6*th* Divide 9$\frac{1}{2}$ by $\frac{2}{5}$ of $\frac{3}{4}$, *facit* $\frac{380}{12}$, 31 $\frac{2}{3}$.

The reaſon for this rule is, that the numerator divided by the denominator is the expreſs quantity of every fraction; therefore when one fraction is to be divided by another, we take the numerators as the numbers given, which muſt be cleared from being divided by their denominators, that is, we muſt take away thoſe diviſors, according to the laſt example in Multiplication; which is done by multiplying each numerator by the others denominator, the products whereof will be the numbers or numerators, cleared for diviſion, and both thereby increaſed in the ſame proportion.

$\frac{1}{2})\frac{3}{4}(\frac{6}{4}=1\frac{1}{2}$

Suppoſe $\frac{3}{4}$ to be divided by $\frac{1}{2}$, as at the ſide, here 3 is to be divided by 1 ; by the fractions $\frac{3}{4}$ and $\frac{1}{2}$, are the quotients of 3 divided by 4, and of 1 divided by 2 ; therefore, to clear each from being quotients, and to increaſe them in a like proportion, we multiply 3 by 2, and 1 by 4 ; which is really multiplying both the given fractions by 8, and produces $\frac{6}{4}$ equal to 1 $\frac{1}{2}$.

This ſhews, that when a fraction, or any quantity, is divided by a proper fraction, the quotient muſt be greater than the dividend, in the ſame proportion as the diviſor is leſs than an unit.

If a mixed number is given to be divided by a whole number, the integral part of the dividend may firſt be divided ; and, if any thing remains, multiply it by the denominator of the fraction, and add the numerator of the product for a numerator of the fractional part of the quotient, and place the product of the diviſor ſo multiplied for a denominator ; but, if ſuch numerator is greater than the denominator, the exceſs is to be the numerator, and 1 added to the integral part of the quotient.

The DIRECT RULE *of* THREE
IN
VULGAR FRACTIONS.

AFter the numbers are prepared by reduction, ſtate and work the queſtion by the rules given in whole numbers.

EXAM-

EXAMPLES.

1ſt. What will $\frac{1}{2}$ a pound of ſnuff coſt, if 22 ℔ $\frac{1}{4}$ of the ſame come to 7 *l.* $\frac{1}{3}$?

$$\begin{array}{ccc} \text{℔.} & l. & \text{℔.} \\ \textit{If } 22\tfrac{1}{4} \textit{ coſt } & 7\tfrac{1}{3}, \textit{ what will } & \tfrac{1}{2}\,? \end{array}$$

Prepared by the 2d rule of reduction, it will ſtand thus:

$$\textit{If } \tfrac{89}{4} \;.\;.\; \tfrac{22}{4} \;.\;.\; \tfrac{7}{2}.$$

then the ſecond and third numbers multiplied together produce $\frac{22}{6}$; which divided by $\frac{89}{4}$, gives $\frac{88}{534}$; which valu'd by the 7th rule of reduction, is 3*s.* 3*d.* 2 *qu.* $\frac{108}{534}$.

2d. Bought 87 gallons and an $\frac{1}{2}$ of brandy for 40 *l.* at the ſame rate, what would $\frac{1}{2}$ a gallon coſt?

$$\begin{array}{ccc} \textit{Gall.} & l. & \textit{Gall.} \\ \textit{If } 87\tfrac{1}{2} \textit{ coſt } & 40, \text{ what will } & \tfrac{1}{2}\,? \\ \textit{prepared } \tfrac{175}{2} \text{ —— } & \tfrac{40}{1} \text{ ———— ———— } & \tfrac{1}{2}\,? \end{array}$$

$$\begin{array}{lcccc} & l. & s. & d. & qu. \\ \textit{then } \tfrac{40}{1} \times \tfrac{1}{2} = \tfrac{40}{2} \div \tfrac{175}{2} = \tfrac{80}{350} = \tfrac{8}{35}, \text{ or} & 4 & .\;6 & . & 3\tfrac{3}{7}. \end{array}$$

Another RULE.

When your queſtion is ſtated, and your numbers prepared, multiply the *denominator* of the firſt number and the *numerators* of the ſecond and third continually, and place the product for a *numerator*. Then multiply the *numerator* of the firſt number and the *denominator* of the ſecond and third continually, and place the product for its *denominator*; the new fraction ſo found, is the anſwer of the queſtion. Take the ſtating of the laſt queſtion for an

EXAMPLE.

If $\frac{175}{2} \,..\, \frac{40}{1} \,..\, \frac{1}{2}$ *facit* $\frac{80}{350}$.

For 2, the *denominator* of the firſt number, multiply'd by 40 and 1, the *numerators* of the ſecond and third numbers, is 80.

And 175, the *numerator* of the firſt, multiply'd by 1 and 2, the *denominators* of the ſecond and third, is 350. Theſe products, placed fractionwiſe, make $\frac{80}{350}$, as before.

3*d*. What will 82 *C.* of ſugar come to, if $\frac{3}{4}$ of $\frac{3}{8}$ of a *C.* coſt $\frac{9}{10}$ of a pound Sterling ?

$$\begin{array}{l}
\qquad\quad C. \qquad\quad l. \qquad\qquad\qquad\quad C. \\
\textit{If } \frac{3}{4} \textit{ of } \frac{3}{8} \textit{ coſt } \frac{9}{10}\textit{, what will } 82\,? \\
\qquad\quad \frac{9}{32} \,..\, \frac{9}{10} \,..\, \frac{82}{1}
\end{array}$$

Facit $\frac{23616}{90}$, or 262*l.* 8*s.*

More EXAMPLES.

What will $\frac{1}{4}$ of an ounce of ſnuff coſt, if the ℔. comes to 8*s.* $\frac{1}{3}$?

Anſwer, 1 *d.* 2 *qua.* $\frac{1}{4}$.

At 1*d.* $\frac{1}{2}$ *per* ounce, what will 5 *C.*$\frac{1}{2}$ come to ?

Anſwer, 61*l.* 12*s.*

The INDIRECT RULE *of* THREE
IN
VULGAR FRACTIONS.

HERE alſo you have only to obſerve the directions given in whole numbers :

Or this RULE.

After the numbers are ſtated and prepared, multiply the *numerators* of the firſt and ſecond numbers and the *denominator* of the third continually for a *numerator* ; and the *denominators* of the firſt and ſecond numbers, and the *numerator* of the third continually for a *denominator* ; the fraction ſo found is the *Anſwer*. See both ways in the example.

EXAMPLE.

How many yards of ſtuff, $\frac{1}{2}$ yard wide, are equal to $36\frac{1}{4}$ yards of $\frac{3}{4}$ wide ?

$$\textit{If } \frac{3}{4} \text{———} 36\frac{1}{4} \text{———} \frac{1}{2}$$

$$\textit{Prepared } \frac{3}{4} \;\ldots\ldots\; \frac{145}{4} \;\ldots\; \frac{1}{2}$$

$$\frac{3}{4} \times \frac{145}{4} = \frac{435}{16},\ \frac{435}{16} \div \frac{1}{2} = \frac{870}{16},\ \textit{or } 54\frac{3}{8}.$$

So alſo 3, 145, and 2, the *numerator* of the firſt and ſecond numbers, and the *denominator* of the third, multiplied together, is 870.

And 4, 4, and 1, the *denominators* of the firſt and ſecond, and *numerator* of the third, multiplied together, produce 16.

Theſe numbers, fractionally placed, make $\frac{870}{16}$, as before.

More E X A M P L E S.

If *A.* lends *B.* 35*l.* $\frac{1}{3}$ for 6 months $\frac{1}{2}$, how long may he keep 10 *l.* $\frac{1}{4}$ of *B*'s, to requite himſelf?

Anſwer, 22 months, $\frac{50}{123}$.

If *A.* keeps 100'. $\frac{1}{6}$ of B's 4 months, $\frac{1}{3}$, what ſum muſt *A.* lend *B.* for 2 years $\frac{1}{2}$ to requite him?

Anſwer, 14*l* $\frac{253}{540}$.

The COMPOUND RULE *of* THREE IN VULGAR FRACTIONS.

TO the Rules before given page 42, let it be obſerved, that where any of the terms given is a fraction (or a mixed number, which muſt be put down as an improper fraction) in the operation, the denominator will be a factor contrary to the numerator; and when an equation is made for the quotient Q, where the ſeveral factors of the dividend are put as a numerator, and the ſeveral factors of the diviſor as a denominator; then whatever factors in each are alike, they may be ſtruck out, and for thoſe equally diviſible take their quotients, whereby the operation will frequently be very much ſhortned.

EXAMPLE.

If 264 men, in $5\frac{3}{4}$ days working $10\frac{1}{2}$ hours each, dig a trench $234\frac{2}{3}$ yards long, $3\frac{1}{4}$ wide, and $2\frac{7}{8}$ deep; in how many days working, $9\frac{3}{4}$ hours each, will 25 men dig a trench $337\frac{1}{2}$ yards long, $4\frac{1}{3}$ wide, and $3\frac{3}{4}$ deep?

	men	*days.*	*hours.*	*long.*	*wide.*	*deep.*
If	264	$\frac{23}{4}$	$\frac{21}{2}$	$\frac{704}{3}$	$\frac{13}{4}$	$\frac{23}{8}$
	25	Q	$\frac{39}{4}$	$\frac{675}{2}$	$\frac{13}{3}$	$\frac{15}{4}$

Here as in *Ex.* 5. page 44, the trenches are the produced terms, and the men, days and hours the producing; therefore

$$Q=\frac{675\times13\times15\times264\times23\times21\times4\times3\times4\times8}{704\times13\times23\times25\times39\times4\times2\times2\times3\times4}=\frac{27\times21}{4}$$

$$=\frac{567}{4}=141\tfrac{3}{4}\ \textit{days.}$$

THE
ARITHMETIC
OF
DECIMAL FRACTIONS.

THIS part of Arithmetic is very compendious and eaſy in operation, and therefore very uſeful, eſpecially in calculations of Intereſt, valuing annuities, &c. But, by reaſon ſome fractions cannot be exactly expreſſed by decimal parts, it may not always be convenient to work with them. Mr. *Cunn* hath, in his late treatiſe on fractions, very curiouſly ſhewn how they may be uſed with leaſt loſs; but, ſince ſuch exactneſs deſtroys the brevity, I cannot but think, where great perfection is required, vulgar fractions preferable.

CHAP. I.

Of NUMERATION, ADDITION, SUBTRACTION, MULTIPLICATION, *and* DIVISION *of* DECIMAL FRACTIONS.

NUMERATION.

IN DECIMAL FRACTIONS, the figures expreſſed with a point or comma prefixed is the numerator, and 1 with as many cyphers as the numerator hath places is always the denominator, and therefore known without being ſet down; thus, ,5 ſtands for $\frac{5}{10}$, ,25 for $\frac{25}{100}$, ,123 for $\frac{123}{1000}$, *&c.* wherein we imagine the integer or unit (whether it be time, coin, weight, meaſure, *&c.* as a year, a pound ſterling, a pound weight, a buſhel, a yard, a foot, *&c.*) to be divided into 10, 100, 1000, *&c.* equal parts at pleaſure; for dividing it firſt into 10 equal parts, and then each of thoſe into 10 leſſer equal parts, will divide the unit into 100 equal parts; and if again we divide each of thoſe hundredths into 10 equal parts, the unit will be divided into 1000 equal parts, and ſo on infinitely.

The different value of the ſeveral places will more plainly appear from their explication under the following number.

Integral Parts.						Fraction Parts.					
6	5	4	3	2	1,	2	3	4	5	6	7,
C Thouſands.	X Thouſands.	Thouſands.	Hundreds.	Tens.	Units.	Tenths.	Hundredths.	Thouſands.	X Thouſands.	C Thouſands.	Millions.

From whence it is evident,

That as the ſignificant figures or parts in the whole numbers increaſe by a ten-fold proportion towards the left-hand from the units place; ſo, in Decimal Fractions, they decreaſe in the ſame proportion towards the right; therefore cyphers at the left-hand in a decimal fraction decreaſe the value in a tenfold proportion, by removing thoſe ſignificant figures farther from the unit's place.

$$\text{Thus}\left\{\begin{array}{l} ,5 \text{ is five tenth parts} = \frac{5}{10} = \frac{1}{2} \\ ,05 \text{ is five hundredths} = \frac{5}{100} = \frac{1}{20} \\ ,005 \text{ is five thouſandths} = \frac{5}{1000} = \frac{1}{200} \end{array}\right\}\text{of an unit.}$$

But cyphers at the right-hand of a decimal fraction alter not its value, becauſe the places of the ſignificant figures thereof are not thereby changed; for ,5 ,50 ,500 *&c.* are each $\frac{5}{10}$ of an unit, the 5 in each being in the firſt place of fractions, *viz.* the place of tenths.

Theſe things underſtood, the reſt is eaſy.

ADDITION.

HAVING ſet down your propoſed numbers, units under units, and fraction places under places of like value, add them as if they were all whole numbers, ſeparating ſo many fractional places in the total, as were in that given number which had the moſt.

EXAMPLES.

Add 5,42, 38,583, 421,06, and ,8345 together.

Thoſe numbers, 5 ,42
rightly placed, 38 ,583
will ſtand thus : 421 ,06
,8345

Total, 465 ,8975

l. l. l.
What's the ſum of 4,06, 21,734, and 523,8642
21,734
4,06

Anſwer, 549,6582*l.*

SUBTRACTION.

HAVING placed the numbers as directed in addition, work ſtill as if they were all whole numbers.

EXAMPLES.

l. l.
From 839,38275 take 10,49561
10,49561

828,88714 *Remainder.*

Yards. Yards.
Take 23,279 *from* 451,15842
23,279

Remains, 427,87942

MULTIPLICATION.

HERE the numbers are to be ſet down, without regard to the value of their places, and worked in all reſpects the ſame as in whole numbers; only obſerve to ſeparate ſo many places of fractional parts at the right-hand in the product, as were in both multiplicand and multiplier.

EXAMPLES.

Multiply	2,735641	
by	,54382	
		5471282
	21	885128
	82	06923
	1094	2564
	136 8	205
	1,4876	9628862

Multiply	73,82564365	
by	8,7434592	
		14765128730
		66443079285
	3	6912821825
	29	530257460
	221	47693095
	2953	0257460
	51677	950555
	590605	14920
Product	645,491	503167514080

A compendious way to multiply numbers that have many fractional places; by omitting all the figures to the right of the perpendicular lines in the examples above.

RULE.

Place one of the numbers juſt as it is given, for a multiplicand, and under that fractional place which you would have be the laſt in the product, ſet the unit's place of your other number; then reverſing all the other figures, multiply with them in their order, beginning always with that figure of the multiplicand, which ſtands over the number you work with; having alſo reſpect to the increaſe you think would have been brought from the omitted figures on the right-hand, had they been multiplied.

Note, The uſual allowance for ſuch omitted figures is as follows, *viz.* If the next figure on the right-hand of that you begin with in the multiplicand, multiply'd into the figure of the multiplier you are working with, gives a product between 5 and 15, carry 1; if the product be above 15, and leſs than 25, carry 2; and if it ariſe to any number between 25 and 35, carry 3, *&c.*

To prove the certainty of the rule, we will take the ſame example before given.

EXAMPLES.

Multiply 2,735641 by ,54382, and produce only four places of decimals.

2,735641
28345,0

13678 *the Increaſe here carried is* 3 *for* 5×6
1094 *the Increaſe here is* 2 *for* 4×5
82 *the Increaſe here is* 1 *for* 3×3
22 *the Increaſe here is* 6 *for* 8×7

1,4876

Note, There being no units in the multiplier, its place is ſupplied with a cypher.

Multiply 73,82564365 by 8,7434592, and produce but 3 places of decimal fractions.

73,82564365
2954347,8

590605
51678
2953
222
29
4 *the Increaſe carried for* 5×7.

645,491

Note, To multiply any decimal fraction by 10, 100, 1000, *&c.* is only to remove the mark of ſeparation ſo many places towards the right-hand as there are cyphers.

$$\text{Thus, } 8{,}3564537 \text{ multiplied by} \left\{\begin{array}{l}10,\\100,\\1000,\\10000,\end{array}\right\} \text{is} \left\{\begin{array}{l}83{,}564537\\835{,}64537\\8356{,}4537\\83564{,}537, \ \&c.\end{array}\right.$$

DIVISION.

THIS rule alſo being work'd the ſame as in whole numbers, the only difficulty is to value the quotient; which may be done by either of the following general rules, *viz.*

RULE 1.

The firſt figure in the quotient is always to ſtand in the ſame place, with that figure of the dividend, which anſwers, or ſtands over the place of units in the diviſor, Or,

RULE 2.

The quotient muſt always have ſo many fractional places, as the dividend has more than the diviſor.

Note, 1*ſt.* If the diviſor and dividend have both the ſame number of fraction places (as in the ſecond example) the quotient will be all a whole number.

2*d.* If the dividend hath not ſo many places of fractions as are in the diviſor, as in example the 3d.) ſo many cyphers muſt be annexed to the dividend, as will make them equal; and the quotient will then, as before, all be a whole number.

3*d.* But if, when diviſion is done, the quotient has not ſo many figures as it ſhould have fraction places (as in the laſt example) ſo many cyphers muſt be prefix'd, as there are places wanting.

EXAM-

EXAMPLES.

1ſt. Divide 10,55439 by 4,569.

4,569)10,55439(2,31

14163

4569

...

Here, according to the firſt rule, 4, the unit's place of the diviſor, ſtanding under units, or the firſt place of whole numbers in the dividend; 2, or firſt figure plac'd in the quotient, muſt alſo be made the firſt place of whole numbers, by placing the point of ſeparation immediately after it.

Or, according to the ſecond rule, there being five places of fractions in the dividend, and but three in the diviſor; two places muſt be ſeparated off in the quotient, to make up the number.

Ex. 2d. Divide 85643,825 by 6,321.

6,321)85643,825(13549

22433

.34708

.31032

57485

.596

Here 6, the place of units in the diviſor, ſtanding under 8, the fifth place of the whole numbers in the dividend, 1, the firſt figure of the quotient, muſt alſo ſtand in the ſame place, which makes all the quotient whole numbers.

And ſo it muſt alſo be by the ſecond rule, becauſe the dividend has no more places of fractions than the diviſor.

Ex. 3. Divide 7382,54 by 6,4352.

6,4352)7282,5400(1147

Here there not being ſo many places of fractions in the dividend as in the diviſor, two cyphers are added to make them even; and being ſo, the quotient is all a whole number as before.

Ex. 4. Divide ,08516438 by 423.

423),08516438(,00020133
. . 564
1413
, 1448
. 179

In this example, the quotient conſiſting but of five figures, whereas there ſhould, by the rules, be eight fraction places of decimal parts, three cyphers are prefixed to make up the number, as was directed in *Note* the third.

There is alſo a way of contracting the work of a large diviſion, ſomewhat like that compendious method before taught, of multiplying numbers of many places of decimal fractions, and bringing out only ſo many in the product as are neceſſary for the anſwer required.

RULE.

Having conſider'd, by the firſt rule given for valuing the quotient, in what place the firſt figure thereof ought to ſtand, and accordingly determined how many figures it is neceſſary it ſhould conſiſt of, you may from thence judge whether any, or how many of the right-hand figures of your diviſor may be neglected; ſo that reſerving only ſo many figures of the dividend as are neceſſary for once anſwering the diviſor, leave out the reſt to the right-hand as uſeleſs; and having ſet the proper figure in the firſt place of the quotient, work with it as uſual; then omitting the right-hand figure of your diviſor, ſeek how often the other figures of your diviſor are contained in the remainder; which figure being entered in the quotient, and work'd with as uſual in diviſion, (with regard, however, to the carriage that would have been brought from the omitted figures, as before directed in multiplication) thus continuing to neglect the right-hand figure (putting a point under for ready knowing it) of your diviſor every time you ſeek a new quotient figure, you will ſtill be able to divide the remainder left after the laſt ſubtraction by the diviſor ſo leſſened, till you have brought out the determined number of figures in the quotient.

The following example, which is ſet down both at length, and the contracted way, will make all clear.

EXAM-

EXAMPLE.

```
3,4637528)95,4327|56463275(27,55188
             69275056|6
             ------   ----       3,46375)95,4327(27,5518
             261577|004                  69,2750
             242462|696                  -------
             ------   ----               26,1577
              .19114|3086                24,2463
               17318|7640                -------
               -----  ----                 19114
              .1795|54463                  17319
               1731|87640                  -----
               ----  -----                  1795
                .63|668232                  1732
                 34|6375 28                 ----
                 --  -----                    63
                 29|0307047                   35
                 27|7100224                   --
                 --  -------                  28
                 .1|32068235                  27
                 --  --------                 --
                                              .1
```

And thus are all the figures on the right-hand of the perpendicular line in the example work'd at length, ſav'd by working after this contracted manner.

As multiplying by 10, 100, 1000, *&c.* is only removing the ſeparating point of the multiplicand, ſo many places towards the right-hand as there are cyphers in the multiplier; ſo, to divide by 10, 100, 1000, *&c.* is only removing the mark of ſeparation, after the ſame manner, towards the left-hand.

Thus 83564,537 divided by	10,	is	8356,4537
	100,		835,64537
	1000,		83,564537
	10000,		8,3564537

Theſe examples being, I hope, ſufficient to make diviſion plain, we will now proceed to *Reduction of Decimals.*

CHAP. III.

Of REDUCTION *of* DECIMALS.

THERE are two ſorts of *Reduction of Decimal Fractions*. 1ſt. Changing a vulgar fraction to a decimal. 2. Valuing a decimal fraction by the known parts of an integer.

1ſt. To reduce or change a vulgar fraction to a decimal.

RULE.

Place cyphers at pleaſure after the *numerator*, and divide by the *denominator*.

EXAMPLES.

Reduce $\frac{1}{4}$ to a decimal, 4)1,00(,25 *facit.*

What decimal parts are equal to $\frac{3}{8}$?

8)3,00(,375 *facit.*

And ſo any part of coin, weight, meaſure, time, &c. may be brought into a decimal, by firſt reducing it into a vulgar fraction which is done by placing the known parts of the integer as a denominator under the parts given) then annexing cyphers to the *numerator*, and dividing by the *denominator*, as before directed.

EXAMPLES.

What decimal parts of a pound are equal to 16*s.*?
i. e. $\frac{16}{20}$ 20)16,0(,8 *facit.*
What decimal parts of a ℔. *Troy*, are equal to 9 *oz.*?
i. e. $\frac{9}{12}$ 12)9,00(,75 *facit.*

Note, If the given parts are of ſeveral denominations, they may be reduced either by ſo many diſtinct operations as there are different parts, or by firſt reducing them into their loweſt denomination, and then one diviſion will ſerve. See both ways in the

EXAMPLES.

Reduce 14 *s.* 9 *d.* $\frac{3}{4}$ into decimal parts of a pound Sterling: By the firſt way there will be theſe three vulgar fractions to be changed, $\frac{14}{20}$, $\frac{9}{240}$, and $\frac{3}{960}$.

20)14,00(,70	240),90000(,0375	,7
		,0375
960)3,0000(,0031	1800	,0031
.1200	1200	*facit* ,7406
.240	.00	

s. d.
The other way. 14 $9\frac{3}{4}$ *being reduced to farthings.*
12
177
4
will be 711
which divided by its denominator 960)711,0000(,7406 *is produced as (before.*
.3900
.6000
.240

What decimal parts of a C.*wt*.are equal to 3 *qrs*. 18 ℔.?

By the firſt way :

$\frac{3}{4}$ and $\frac{18}{112}$ 4)3,00(,75
,1607

Anſwer, ,9107

112)18,0000(,1607
.680
..800
16

By the ſecond way.

qrs. ℔.
3 18
28

102

112 *Denominator.*

Anſwer, 112)102,0000(,9107
.120
.800
.16

There is alſo another way of reducing parts of different denominations into decimals of the higheſt integer ; thus,

RULE.

Bring the loweſt parts into decimals of the next ſuperior denomination ; and on the left-hand of the decimal fraction found, place the parts given of that ſaid next ſuperior denomination ; ſo proceeding, till you bring out the decimal parts of the higheſt integer requir'd, by ſtill dividing the product by the next ſuperior *denominator*. For examples, take thoſe before given, *viz.*

Reduce 14*s.* 9*d.* $\frac{3}{4}$ into decimal parts of a pound Sterling.

4	3,
12	9,75
20	14,8125
	,7406

Here the 3 farthings being divided by 4, gives 75, to which the 9*d.* being prefixed, it becomes 9,75 ; which divided again by 12, quotes ,8125 ; before which the 14*s.* being alſo placed, it makes 14,8125 ; which being laſtly divided by 20, the quotient ,7406, is the anſwer.

What decimal parts of a *C.wt.* are equal to 3 *qrs.* 18 ℔?

28	7	18,
	4	2,5714
	4	3,6428
		,9107 *Anſwer.*

Note, Inſtead of dividing by 28, which wou'd be troubleſome ; 'tis better to divide by 7 and 4, as in the example, 4 times 7 being 28.

Thus you ſee how eaſily the decimals, anſwering to any given parts of coin, weight, meaſure, &c. are found ; and after the ſame manner are the following tables formed.

But ſhillings, pence, and farthings, may more readily be reduc'd thus ;

RULE.

For the ſhillings (if their number be even) ſet down their half in the firſt place of decimal fractions, and let the ſecond and third places be filled up with the farthings contain'd in the remaining pence and farthings, always remembring to add one, when the number

ber is or exceeds 25; but if the number of ſhillings be odd, the ſecond place muſt alſo be increaſed by 5.

Thus 8*s.* 5*d.* ½, reduced to a decimal fraction, is ,422; 4 in the firſt place ſtanding for 8*s.* and 22 farthings being 5*d.* ½.

Again, 8*s.* 6*d.* ½, will be ,427.

1 being added to 26, the number of farthings in 6*d* ½, becauſe exceeding 25.

And laſtly, 9*s.* 6*d.* ½, expreſs'd decimally, is ,477.

5 being added to the ſecond place, becauſe the number of ſhillings is odd, 1 ſhilling being ,05.

The ſecond ſort of reduction is, to value a decimal fraction by the known parts of its integer; which is thus done:

RULE.

Multiply the decimal fraction given by the known parts of the next inferior denomination, and ſeparating the product as directed in multiplication, then multiply the decimal fraction of the product by the known parts of the next inferior denomination; thus proceeding till you have brought it into the leaſt known parts of the integer, the figures ſtanding on the left of the ſeparating points will be the parts required.

EXAMPLES.

Ex. 1. What's the value of ,7691, the integer a pound ſterling? (*i. e.*) What ſhillings, pence, and farthings are equal to ,7691 parts of a pound ſterling?

	,7691	*parts of a pound*
multiplied by	20	*the ſhillings in* 1*l.*
produce ſhillings	15,3820	*and parts of a ſhill.*
which multiplied by	12	*the pence in* 1 *ſhill.*
make pence	4,5840	*and parts of a penny.*
which again multiplied by	4	*the farthings in* 1*d.*
give farthings	,3364	*and parts of a farth.*

So that the value of ,7691 decimal parts of a pound ſterling is 15*s.* 4*d.*½.

Ex. 2. What ounces, penny weights, and grains, are equal to ,84362 decimal parts of a pound *Troy?*

,84362 *parts of a pound* Troy,
multiplied by 12 *the ounces in one pound.*

give ounces 10,12344 *and parts of one ounce.*
multiplied by 20 *the penny weights in* 1 *oz.*

give penny-weights 2,46880 *and parts of* 1 *penny-wt.*
multiplied by 24 *the grains in* 1 *penny-wt.*

187520
93760

Give grains 11,25120 *parts of* 1 *grain.*

The Anſwer *therefore is* 10 *oz.* 2 *dwts.* 11 *grs.*

But the decimal parts of a pound ſterling may be thus valued at ſight:

RULE.

The figure ſtanding in the firſt place of decimal parts doubled, gives ſhillings; but if the figure in the ſecond place is or exceeds 5, one more muſt be added to their number; the ſecond figure, (if under 5) or its exceſs, (if above 5) joined with the third, are ſo many farthings; only remember to abate 1, if their number amounts to near 25; or 2, if near 50.

The reaſon of this abatement is, that as 1000 is the *denominator* of every decimal fraction conſiſting of 3 places, ſo conſequently by reckoning the figure ſtanding in the ſaid third place, as ſo many farthings, we thereby allow 1000 farthings to the pound; whereas indeed there are but 960: The overplus therefore being 40 in 1000, is 4 in a 100, or 1 in 25, &c. according to the direction above.

EXAMPLES.

Ex. 1. What's the value of ,375 parts of a pound ſterling?

Anſwer, 7*s.* 6*d.*

The 3 doubled is 6 ſhillings; to which 1 *s.* being added for the 5, in the ſecond place, makes it 7*s.* and 2 remaining, join'd with the 5 in the 3d place, being accounted 25 farthings, from which 1 being deducted, there remain 24 farthings, or 6 pence.

Ex 2. What's the value of ,846 parts of a pound?

Anſwer, 16*s.* 11*d.*

The 8 doubled, making 16*s.* and 2 abated from the 46 farthings, leaves 44 farthings, or 11*d.*

Thus are ſhewn the ways both of finding the decimal fractions anſwerable to any given parts of coin, weight, meaſure, &c or the value of any given decimal in the known parts of its integer.

I have likewiſe annexed TABLES ready calculated by the foregoing rules, anſwering to the ſame ends.

The uſe of which is very plain; one column in each table containing the number of ſhillings, ounces, drachms, pints, buſhels, days, hours, or any thing elſe you may want; and againſt the ſaid number, in the other column, the decimal parts anſwering: Thus againſt 3 pence in the firſt table, is found ,0125, and againſt 40 days in the fourth table ,109589.

But if the number you want is not to be ſound in the ſaid tables (many compound numbers having been omitted, to reduce them into ſo narrow a compaſs) you muſt add the parts of ſuch two or three numbers together, as will compoſe it, and the total will be the decimal fraction ſought. Thus, if the decimal fraction anſwering to 17 penny-weights, the integer a pound *Troy*, were to be ſought in the 2d table, ,04166, the decimal fraction anſwering to 10 penny-weights, and ,029166, that anſwering to 7 penny-weights, and added together, the total ,070832, will be found the anſwer; or, if in table the fourth, you would find the decimal of 352 days,

add	,821918	*the decimal fractions of*	300	*Days.*
	,36986		50	
	,005479		2	
total	,964383	*anſwering*	352	*Days.*

And ſo of any other.

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Decimal Tables of Coin & Weight

Table 1st.

Coin
1 £ ster: y^{e} Integer

S	Dec:	S	Der:
19	,95	9	,45
18	,9	8	,4
17	,85	7	,35
16	,8	6	,3
15	,75	5	,25
14	,7	4	,2
13	,65	3	,15
12	,6	2	,1
11	,55	1	,05
10	,5		

Pence	Decim:
11	,045833
10	,041666
9	,0375
8	,033333
7	,029166
6	,025
5	,020833
4	,016666
3	,0125
2	,008333
1	,004166

Farthings	Decim:
3	,003125
2	,002083
1	,001042

Table 2d.

Troy Weight
1 lb. the Integer

Ounces	Decim:
11	,916666
10	,833333
9	,75
8	,666666
7	,583333
6	,5
5	,416666
4	,333333
3	,25
2	,166666
1	,083333

Note. This Table of Oz: will also serve for Inches Months or Doz:

Penny w^{t}.	Decim:
10	,041666
9	,0375
8	,033333
7	,029166
6	,025
5	,020833
4	,016666
3	,0125
2	,008333
1	,004166

Grains	Decim:
20	,003472
10	,001736
9	,001562
8	,001389
7	,001216
6	,001042
5	,000868
4	,000694
3	,000521
2	,000347
1	,000173
$\frac{1}{2}$	,000086

Table 3d.

Avoirdupoise
112 lb. y^{e} Integer.

Quarters	Decim:
3	,75
2	,5
1	,25

Pounds	Decim:
20	,178571
10	,080286
9	,080357
8	,071428
7	,0625
6	,053571
5	,044643
4	,035714
3	,026786
2	,017857
1	,008928

Ounces	Decim:
10	,005580
9	,005022
8	,004464
7	,003006
6	,003348
5	,002790
4	,002232
3	,001673
2	,001116
1	,000558

Drams	Decim:
10	,000348
9	,000313
8	,000279
7	,000244
6	,000209
5	,000174
4	,000139
3	,000104
2	,000069
1	,000034
$\frac{1}{2}$	,000017

Webſter ſcripſit. Bickham Sculpſit.

Decimal Tables of Measure and Time

Table 4th.

Time
1 year ye Integer

Days	Decim:
300	.821918
200	.547945
100	.273973
90	.246575
80	.219178
70	.191781
60	.164383
50	.136986
40	.109589
30	.082192
20	.054794
10	.027397
9	.024657
8	.021918
7	.019178
6	.016438
5	.013699
4	.010959
3	.008219
2	.005479
1	.002739

Webster script.

Table 5th.

Measure
Liquid. Dry.
Integer
1 Gall: 1 Quarter

Pints	Decim:	Bush
7	.875	7
6	.75	6
5	.625	5
4	.5	4
3	.375	3
2	.25	2
1	.125	1

qr. Pts	Decim:	Pecks
3	.09375	3
2	.0625	2
1	.03125	1

Decim:	qrs Pecks
.023437	3
.015625	2
.007812	1

Decim:	Pints
.005859	3
.003906	2
.001953	1

Cloth Measure
1 Yard ye Integer

Quarters	Decim:
3	.75
2	.5
1	.25

Nails	Decim:
3	.1875
2	.125
1	.0625

Table 6th.

Time
1 Day ye Integer

Hours	Decim:
20	.833333
10	.416666
9	.375
8	.333333
7	.291666
6	.25
5	.208333
4	.166666
3	.125
2	.083333
1	.041666

Minutes	Decim:
50	.034722
40	.027777
30	.020833
20	.013888
10	.006944
9	.00625
8	.005555
7	.004861
6	.004166
5	.003472
4	.002777
3	.002083
2	.001388
1	.000694

Bickham sculpt.

BRITISH MUSEUM
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CHAP. III.

Of the RULE *of* THREE.

THIS *Rule of* PROPORTION having been already ſufficiently explained, both as to its nature and operation, in *Vulgar Arithmetic*, 'tis needleſs to repeat here what has been there ſaid. Only therefore obſerve, that inſtead of preparing your numbers, by reducing them into their loweſt denomination, you muſt now bring their fractional parts into decimals. See the examples.

EXAMPLES.

Ex. 1. If the price of 26 yards and ½ of drugget is 3*l.* 16*s.* 3*d.* what will 32 yards ¼ come to?

The fractional parts of the numbers being reduced to decimals, and the queſtion ſtated as uſual, the work will ſtand as follows:

Yards. *l.* *Yards.*
If 26,5—3,8125—32,25
32,25
190625
76250
76250
114375
26,5)122,953125(4,63974
1695
1053
2581
1962
1075
..15

The quotient valued by the ſhort rule for money, makes 4 l. 12 s. 9 d. ½.

Ex. 2. What will the pay of 540 men amount to, at 1*l.* 5*s.* 6*d.* *per* man?

Man. *l.* *Men.*
If 1———1,275———540
540
51000
6375

Anſwer, 688,500 *or* 688*l.* 10*s.*

For more examples, take thoſe given in *Vulgar Arithmetic.*

The Indirect RULE *of* THREE.

THE Indirect *Rule of Three* is alſo the ſame here as in *Vulgar Arithmetic*, differing only in the preparation of the numbers.

EXAMPLE.

What muſt the penny-loaf weigh, when wheat is 10*s*. the buſhel, if, when at 6*s*. 5*d*. *per* buſhel, the penny-loaf weighs 5 *oz*. $\frac{1}{2}$?

If ,32083 ——— 5,5 ——— ,5
5,5
160415
160415
,5)1,764565(*oz. dwts. grs.*

Anſwer, 3,52913 *oz.* = 3 .. 10 .. 14 nearly.

Note, Where the laſt figure of a Decimal Fraction would recur infinitely, and it be required to have the operation exact, annex the Vulgar Fraction in that place, *viz*. that figure numerator with 9 denominator or their equal, and in any multiplication thereof, add ſuch part of the multiplier in the loweſt place to make up the product. Suppoſe in the example,

l.   *l.*   *oz.*   *l.*
If $,3208\frac{3}{9}$ = $,3208\frac{1}{3}$ —— 5,5 —— ,5
5,5
16040
16040
$18\frac{1}{3}$
,5)$1,76458\frac{1}{3}$ *oz. dwts. grs.*

Anſwer, . $3,5291\frac{2}{3}$ *oz.* = 3 .. 10 .. 14 more near.

Mr. *Gardiner* has ſhewn, in the explication of his Tables of Logarithms, a very eaſy method of getting the logarithm of any number whoſe laſt expreſſed figure, or figures, would recur infinitely perfect in the loweſt place of the tables, upon the foundation of that figure or figures being numerator with 9, 99, 999, *&c*. denominators, that is, with as many nines as there are recurring figures.

The Double or Compound RULE *of* THREE.

IF the carriage of ½ a *C. wt.* 40 miles comes to 6*d.* what will be the charge of carrying 16 *C.*¼ 100 miles?

First, *If* ,5—,025—16,25
,025
8125
3250
,5)40625(

Miles. *Miles.*
Then, *If* 40———,8125———100
100
40)81,2500(

Answer, 2,03125*l.*=2*l.* 0*s.* 7*d.*½.

Or thus,

C. *l.* C.
If 5———025———16,25
40 *Miles.* 100 *Miles.*
20,0 1625,00
,025
8125
3250
20)40,625

Answer, 2,03126*l.*=2*l.* 0*s.* 7*d* ½

Or thus,

C. *Miles.* *l.*
If ,5 carried 40 comes to ,025

$$Q=\frac{,025\times100\times16,25}{,5\times40}=\frac{,05\times10\times16,25}{4,}=2,03125$$

CHAP.

CHAP. IV.

Of PRACTICE.

THE fractions of ſome queſtions in this rule may with advantage be worked decimally ; as,

1ſt. When the given price is juſt two ſhillings, 'tis only ſeparating the right-hand figure for a decimal fraction, the reſt are pounds :

EXAMPLE.

5295 ℔. at 2*s.* *per* ℔ *comes to* { 529,5 *or* 529*l.* 10*s.*

2*d.* When the price is any *aliquot* part of 2*s.* it is only ſeparating the right-hand figure, as before, and then dividing the number by ſuch part.

EXAMPLE.

oz. *d.*
258,3 at 8 *per oz.*

8*d* $\frac{1}{3}$86,1=86*l.* 2*s.* *Anſwer.*

3*d.* When the given price can be divided into *aliquot* parts of two ſhillings, ſeparate as before, and take ſuch parts.

EXAMPLE.

l. *d.*
563,8 at 9 *per* ℔.
d.
6¼ 140,95
3½ 70,475

Anſwer, 211,425=211*l.* 8*s.* 6*d.*

4th. When the price is any number of ſhillings whatſoever, 'tis only multiplying by their half (which is the decimal fraction for them) and the product is the anſwer.

EXAMPLES.

℔. *s.*
7296 at 12 *per* ℔.
,6

Anſ. 4377,6=4377*l.* 12*s.*

oz. *s.*
8264 at 15 *per oz.*
,75

41320
57848

Anſwer, *l.*6198,00

5th. The required price of any quantity may be found, by multiplying the ſaid quantity by the decimal fraction anſwering to the price given.

EXAMPLE.

℔. *l.* *s.* *d.*
8397 at 5 . 13 . 6 *per* ℔.
5,675

41981
587795
50382
41985

*Anſ.*47652,975=47652*l.* 19*s.* 6*d.*

6th. If there are fractional parts in the given quantity, they muſt be reduced to decimals, as well as the fractional parts of the price.

EXAMPLES.

℔. *s.*
368½ at 13 *per* ℔.
368.5
,65
18425
22110
Anſwer, 239,525
Equal to 239*l.* 10*s.* 6*d.*

C. *qrs.* ℔. *l.* *s.* *d.*
7368 . 3 . 14 at 1 . 12 . 6
7368,875
1,625
36844375
14737750
44213250
7368875
Anſwer, 11974,421875
Equal to 11974*l.* 8*s.* 5*d.*¼.

The queſtions propoſed in *Vulgar Arithmetic*, may ſerve for more examples.

CHAP. V.

Of INTEREST *and* REBATE.

THESE rules having been already ſufficiently explain'd in *Vulgar Arithmetic*, it only now remains to ſhew how much more advantageouſly they may be work'd by decimal fractions. And, *firſt*,

SIMPLE INTEREST.

1ſt. THE annual intereſt of any ſum of money is found by only multiplying the given principal by the intereſt of one pound for a year; which,

$$\text{at}\left\{\begin{matrix}4\\5\\6\end{matrix}\right\}\textit{per Cent.}\ \text{is}\left\{\begin{matrix},04\\,05\\,06\end{matrix}\right.$$

found by dividing the rate of intereſt by 100.

EXAMPLES.

l. l.

1ſt. What's a year's intereſt of 536 at 5 *per Cent?*

,05

Anſwer, 26,80=26*l.* 16*s.*

2*d.* What will the intereſt of 2367*l.* come to in a year, at 6*l. per Cent. per Annum?* ,06

Anſwer, 142,02*l.*=142*l.* 0*s.* 5*d.*

2*d.* Thus a year's intereſt of any ſum being found, 'tis only multiplying that ſum by 2, 3, 4, &*c.* and the product is the intereſt of the ſaid ſum, for 2, 3, 4, or more years.

EXAMPLE.

l.

What's the intereſt of 623 for 3 years, at 4 *per Cent?*

,04

24.92 *Intereſt for* 1 *year.*

3

Anſwer, 74,76=74*l.* 15*s.* 2*d.* ½ *for* 3 *years.*

3*d.* Again, If the time required be months, 'tis only dividing the year's amount by ſuch parts as the given months are of a year.

EXAM-

EXAMPLES.

Ex. 1*ſt*. What is the intereſt of 539*l*. for four months, at 5*l*. *per Cent. per Annum?*

l.
539
,05

26,95 *Intereſt for a year.*

Months.
$4\frac{1}{3}$ 8,9833=8*l*. 19*s*. 8*d*. *Anſwer.*

Ex. 2*d*. What will the intereſt of 729*l*. 10*s*. 6*d*. come to in a year and 5 months, at 6 *per Cent. per Annum?*

Note, If there are any odd ſhillings, pence, and farthings, in the principal ſum, they muſt be reduced to decimals; as in this example.

729,525
,06

Months	43,77150	*Intereſt for a year.*
$4\frac{1}{3}$	14,590,0	*Intereſt for* 4 *months.*
$1\frac{1}{4}$	3,647625	*Intereſt for* 1 *month.*
Anſwer,	62,00962;	*Total Intereſt.*

or
62*l*. 0*s*. 2*d*. $\frac{1}{4}$

4*th*. But if it be required to find the intereſt of a ſum of money for any number of days, you muſt either multiply the years amount by the number of days propoſed, and divide the product by 365, or (which will be ſometimes ſhorter) divide the year's amount by 365, and multiply the quotient by the number of days propoſed.

EXAMPLE.

What intereſt will 563*l.* amount to in 126 days, at 6*l. per Cent. per Annum?*

```
      l.
     563
     ,06
    -----
    33,78          365)33,78000(,09254
     126               ---------
    -----               .. 930
   20268               -------
    6756                 2000
   3378                 -----
   -------               1750
365)4256,28(11,661       -----
    -----                 290
   . 606                 ------
   -----                ,09254
   2412                   126
   -----                ------
    2228                 55524
   ------               18508
    . 380               9254
   ------              --------
    . 15               11,66004
```

But for the more ready caſting up the intereſt of money for days, it will be neceſſary to have the intereſt of 1 *l.* for 1 day, at all rates ready calculated, as a table or ſtandard. Thus:

The

The INTEREST *of* 1 l. *for* 1 *Day.*

At	1	*per Cent.* is	,00002739726
	2		,00005479452
	3		,00008219178
	4		,00010958904
	5		,00013698630
	6		,00016438356
	7		,00019178082
	8		,00021917808
	9		,00024657534
	10		,00027397260

The intereſt of 1*l.* for one day, is thus found; the given rate or intereſt of 100*l.* for a year, being divided by 100, quotes the intereſt of 1 *l.* for a year; which again divided by 365, gives the required intereſt of 1*l.* for 1 day; which, when multiplied into both the number of days, and the principal ſum, produces the intereſt for the time required.

EXAMPLE.

What's the intereſt of 560*l.* for 60 days, at 5*l. per Cent. per Annum?*

,0001369863
560 *Principal.*

82191780
6849315

,0767123280
60 *Days.*

Anſwer, 4,6027396800=4*l.* 12*s.* 0*d.* ½.

The year being divided into 12 unequal months, the following verſes may ſerve as a *memorandum* of the number of days in each, *viz.*

Thirty days hath September,
April, June, *and* November;
February *hath twenty-eight alone,*
And all the reſt have thirty-one.

But without any farther trouble, the oppoſite table ſhews, by inſpection only, the number of days from any day in any month, to the ſame day in any other month. As ſuppoſe the number of days, betwixt the 4th of *February* and the 4th of *September*, were required; looking in the column under *February* for *September*, againſt that month will ſtand 212, the number of days betwixt the ſaid times: So again, from the 8th of *June* to the 8th of *October*, appears by the table to be 122 days, that being the number againſt *October*, in the column under *June*.

Note, If the given days are different, ('tis only adding, or ſubtracting their inequality to, or from the tabular number. Thus had the firſt example been from the 4th of *February* to the 8th of *September*, it had been 4 days more than 212, *viz.* 216; or had the time, in the laſt example, been from the 8th of *June* to the 4th of *October*, it had been 4 days leſs than 122, *viz.* 1[illegible].

Note alſo, If the time exceeds a year, 365 days muſt be added. Thus from the 4th of *February* 1758, to the 4th of *September* 1759, will be found to be 577 days, the ſum of 212, and 365. See the Table.

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A Table

Showing the Number of Days from any day in any Month to the same day in any other Month.

From	Jan:	Feb:	Mar:	April.	May:	June.
To	Feb:31	Mar 28	April 31	May.30	June.31	July.30
	Mar:59	April 59	May.61	June.61	July.61	Aug:61
	April 90	May.89	June 92	July.91	Aug:92	Sep:92
	May 120	June 120	July 122	Aug:122	Sep:123	Oct:122
	June 151	July.150	Aug:153	Sep:153	Oct:153	Nov:153
	July.181	Aug:181	Sep:184	Oct: 183	Nov:184	Dec:183
	Aug:212	Sep:212	Oct:214	Nov:214	Dec:214	Jan:214
	Sep:243	Oct:242	Nov:245	Dec:244	Jan:245	Feb:245
	Oct: 273	Nov:273	Dec:275	Jan:275	Feb:276	Mar:273
	Nov:304	Dec:303	Jan:306	Feb:306	Mar:304	April 304
	Dec:334	Jan:334	Feb:337	Mar:334	April 335	May:334
	Jan:365	Feb:365	Mar:365	April 365	May.365	June.365

From	July.	Aug:	Sep:	Oct:	Nov:	Dec:
To	Aug:31	Sep:31	Oct: 30	Nov:31	Dec:30	Jan:31
	Sep: 62	Oct:61	Nov:61	Dec: 61	Jan:61	Feb:62
	Oct: 92	Nov:92	Dec:91	Jan:92	Feb:92	Mar:90
	Nov:123	Dec:122	Jan:122	Feb:123	Mar:120	April 121
	Dec:153	Jan:153	Feb:153	Mar:151	April 151	May 151
	Jan:184	Feb:184	Mar:181	April 182	May.181	June.182
	Feb:215	Mar:212	April 212	May 212	June 212	July.212
	Mar:243	April 248	May.242	June 243	July 242	Aug:243
	April.274	May.273	June 273	July 273	Aug:273	Sep:274
	May.304	June 304	July 303	Aug:304	Sep:304	Oct:304
	June.335	July 334	Aug:334	Sep:335	Oct:334	Nov:335
	July 365	Aug:365	Sep:365	Oct:365	Nov:365	Dec:365

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COMPOUND INTEREST.

1ſt. TO find the amount of any ſum, at any rate of compound Intereſt, for any number of years.

RULE.

Multiply the amount of 1*l.* for a year, (which at 6 *per Cent.* is 1,06, at 5 *per Cent.* 1,05, *&c.*) ſo often into itſelf, as are the number of years propoſed, wanting one; and the laſt product multiply'd by the principal, will give the amount required.

EXAMPLE.

What's the amount of 500*l.* forborn four years, at 4*l.* *per Cent per Annum?*

```
       1,04
       1,04
     ------
     1,0816
       1,04
     ------
   1,124864
       1,04
   --------
 1,1698,856
        500
 ----------
```

Anſwer, 584,92928000*l.*=584*l.* 18*s.* 7*d.*

And thus is the firſt intereſt-table (which ſhews the amount of 1*l.* for any number of years, under 33, at the rates of 5 and 6 *per Cent. per Ann.*) form'd.

The uſe of which is plain and eaſy; for multiplying the figures ſtanding againſt the number of years required, and under the given rate, by the propoſed principal, the product is the amount deſired.

Thus, if the amount of 40*l.* in 20 years, at 5*l. per Cent. per Annum,* were required, 2,65329, the tabular number multiplied by 40*l.* the principal ſum, gives 106,13160, or 106*l.* 2*s.* 7*d.*$\frac{1}{2}$, the *Anſwer*. And ſo of any other.

2*d.* To find the amount of any annuity, or yearly penſion, forborn any number of years whatſoever, at any rate of compound intereſt.

RULE.

Multiply the firſt yearly payment by the amount of 1*l.* for 1 year, and to the product add the ſecond yearly payment, the ſum is the amount in 2 years; which, multiply'd again by the ſaid amount, the product, with the addition of the third yearly payment, is the amount for 3 years, &c.

EXAMPLE.

What will a penſion of 30*l. per Annum* amount to, being foreborn 4 years, at 5*l. per Cent. per Annum?*

	l.	
Firſt yearly payment	30	
multiply'd by	1,05	
	31,50	
Second yearly payment added	30	
gives	61,50	*the amount in 2 years.*
	1,05	
	64,5750	
Third yearly payment	30	
	94,5750	*amount in 3 years.*
	1,05	
	99,303750	
Fourth yearly payment	30	
	129,303750	*amount in 4 years.*

And after this manner is the ſecond intereſt table (which ſhews the amount of 1 *l.* annuity for any number of years under 33, at the rates of 5 and 6 *l. per Cent. per Ann.* compound intereſt) compos'd.

But this ſecond table may more eaſily be formed from the firſt, thus: the firſt line of this ſecond table, or firſt year's amount being known, add of it the firſt line of the firſt table, the ſum is the amount to 2 years, or the ſecond line of this table; to which again adding the ſecond line of the firſt table, you have the third of this, or the amount for three years, *&c.*

EXAMPLE, at 6*l.* *per Cent.*

l.
1, ——— *Firſt year of the ſecond table.*
1,06 ——— *Firſt of the firſt table.*

2,06 ——— *Second year of the ſecond table.*
1,1236 ——— *Second year of the firſt.*

3,1836 ——— *Third year of the ſecond.*
1,19101 ——— *Third year of the firſt.*

4,37461 ——— *Fourth year of the ſecond,* &c.

The uſe of this ſecond table is in the ſame manner as the firſt; as ſuppoſe the foregoing queſtion for an example.

EXAMPLE.

4,31012 { *the annuity of* 1l. *annuity for* 4 *years, at* 5l. per Cent.
multiplied by 30 *the yearly ſum.*

gives 129,30360 *as before.*

REBATE,

REBATE *or* DISCOMPT.

REBATE, according to ſimple intereſt, having been ſufficiently explained in *Vulgar Arithmetic*, I ſhall only here ſet down one example work'd decimally the common way. And, to complete the ſection, add Mr. *Hatton*'s new method of finding the preſent worth and diſcompt of money, as delivered in his *Syſtem of Arithmetic*, pag. 188.

EXAMPLE.

What diſcompt muſt be allowed on a bill of 500*l.* paid 20 days before it is due, rebate at 5*l. per Cent. per Annum*? And what preſent money muſt be paid?

The common way.

Days. — *l.* — *Days.*

If 365——5——20

5

365)100,0(,27397

2700

1450

3550

2650

·95

Then for the diſcompt.

If 100,27397——,27397——500
500

100,27397)136,98500(1,3661 *diſcompt.*

36711030

66288390

61240080

10756980

.629593

Or for the preſent worth.

If 100,27397—100—500
500

100,27397)50000,000(498,6338 *preſent worth.*

98904120

86575470

63562940

33985580

39033890

89516990

9297814

Mr. HATTON'*s New Method.*

For the preſent worth, multiply the days in a year, the principal given, and 100 into each other, for a dividend; and add the product of 365 by 100 to that of the days multiplied into the rate given for the diviſor; ſo the quotient ariſing is the anſwer, Thus in the foregoing example, $365\times500\times100=18250000$ for the dividend, and $365\times100+20\times5=36600$ for the diviſor.

See the work.

366|00)182500|00(498,6338 *preſent worth.*

.3610

3160

2320

240

1420

322

2. *For the diſcompt,* multiply the rate principal, and days given together for a dividend; and proceeding as before directed for a diviſor, the quotient will be the anſwer.

Thus keeping ſtill the ſame example, $5\times500\times20=500060$ for the dividend, and $365\times100+20\times5=36600$ for a diviſor, as before.

See the work.

366|00)500|00(1,3661 *diſcompt.*

1340

2240

2240

.440

74

The truth of both theſe methods appears, not only by their agreement, but alſo by the addition of the preſent worth, and diſcompt, found; which, as near as can be, makes the principal.

Thus 498,6338 *the preſent worth,*
with 1,3661 *the diſcompt.*

makes 499,9999=500*l. the given principal.*

REBATE, *at* COMPOUND INTEREST.

1*ſt.* TO find the preſent worth of any ſum to be paid at any number of years to come, rebate being made at any rate of compound intereſt.

RULE.

Divide the principal continually ſo many times by the amount of 1*l.* for 1 year, as are the number of years propoſed, and the laſt quotient is the anſwer.

EXAMPLE.

What is the preſent worth of 1*l.* to be paid at the end of 6 years, rebate at 6*l. per Cent. per Ann.* compound intereſt?

	l.				*Years.*
The principal	1,	*divi-ded by* 1,06 *gives*	,943396	*preſent worth at the end of*	1
	,943396		,889997		2
	,889997		,839619		3
	,839619		,792094		4
	,792094		,747258		5
	,747258		,704960		6

And thus is the third table made, which ſhews the preſent worth of 1*l.* due at any number of years to come under 33, rebate at 5 and 6 *l. per Cent. per Annum*, compound intereſt; the uſe of which is as the other tables, only by multiplying the preſent worth of 1*l.* by the given principal, and the product is the preſent worth required.

Thus, ſuppoſe in the example above, the principal had been 200*l.* the preſent worth of 1*l.* *viz.* ,704960 multiply'd by 200, gives 140,992000, the preſent worth of 200*l.* and ſo any other.

2*d.* To find the preſent worth of any annuity, or yearly penſion, to continue any number of years; rebate being made at any rate, *per Cent. per Ann.* compound intereſt.

RULE.

Find by the foregoing rule the preſent worth of the propos'd annuity, for 1, 2, 3, or ſo many years as are demanded; and the ſum of thoſe reſpective preſent worths will be the value of the annuity; that is, the firſt and ſecond of thoſe values, added together, will be the preſent worth for two years; and the firſt, ſecond, and third, added for three years, *&c.*

EXAMPLE.

What's the preſent worth of an annuity of 50*l.* to continue four years, rebate at 5*l.* *per Cent. per Annum?*

l.

$$\left.\begin{array}{l}50{,}\\47{,}61904\\45{,}35147\\43{,}19187\end{array}\right\}\begin{array}{c}\textit{divided by } 1{,}05\\ \textit{gives,}\end{array}\left\{\begin{array}{l}47{,}61904\\45{,}35147\\43{,}19187\\41{,}13512\end{array}\right.$$

The total of which is 177,29750 *the preſent* (*worth requir'd.*

And after this manner is made table the fourth, which ſhews the preſent worth of 1*l.* annuity, to continue any number of years under 33, rebate at 5

and

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Decimal Tables,
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TABLE 1st. The Amount of 1£. for any number of Years under 33 at the Rates of 3 & 4 ℘ Ct. ℘ an. Com. Int.

TABLE 2d. The Amount of 1£. Annuity for any number of Years under 33 at the Rates of 3 & 4 ℘ Ct. ℘ an. C. Int.

Rates		Years	Rates	
3	4		3	4
1,03000	1,04000	1	1,00000	1,00000
1,06090	1,08160	2	2,03000	2,04000
1,09273	1,12486	3	3,09090	3.12160
1,12551	1,16986	4	4,18363	4.24646
1,15927	1,21665	5	5,30914	5.41632
1,19405	1,26532	6	6,46841	6,63297
1,22987	1,31593	7	7,66246	7,89829
1,26677	1,36857	8	8,89233	9,21423
1,30477	1,42331	9	10,15910	10,58279
1,34392	1,48024	10	11,46388	12,00610
1,38423	1,53945	11	12,80780	13.48635
1,42576	1,60103	12	14,19203	15.02580
1,46853	1,66507	13	15,61780	16,62684
1,51259	1,73167	14	17,08632	18,29191
1,55797	1,80094	15	18,59891	20,02359
1,60471	1,87298	16	20,15688	21,82453
1,65285	1,94790	17	21,76159	23.69751
1,70243	2,02582	18	23,41443	25.64541
1,75350	2,10685	19	25,11687	27,67123
1,80611	2,19112	20	26,87037	29.77808
1,86029	2,27877	21	28,67649	31,96920
1,91610	2,36992	22	30,53678	34.24797
1,97359	2,46472	23	32,45288	36,61789
2,03279	2,56330	24	34,42647	39,08260
2,09378	2,66583	25	36,45926	41,64591
2,15659	2,77247	26	38,55304	44,31174
2,22129	2,88337	27	40,70963	47,08421
2,28793	2,99870	28	42,93092	49,96758
2,35656	3,11865	29	45,21885	52,96629
2,42726	3,24340	30	47,57542	56,08494
2,50008	3,37313	31	50,00268	59,32833
2,57508	3,50806	32	52,50276	62,70147

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N^{o}.1 Between Pages 202 & 203.

Decimal Tables,

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TABLE 3^{d}. The present Worth of 1£ due at any Number of Years under 33 Rebate at 3 & 4 pC^{t}. p An. Com. Int.

TABLE 4th. The present Worth of 1£ Annuity for any Number of Years under 33. Rebate at 3 & 4 pC^{t}. p Ann.

Rates 3	Rates 4	Years	Rates 3	Rates 4
,970874	,961538	1	0,97087	0,96154
,942596	,924556	2	1,91347	1,88609
,915142	,888996	3	2,82861	2,77509
,888487	,854804	4	3,71710	3,62989
,862609	,821927	5	4,57971	4,45182
,837484	,790314	6	5,41719	5,24214
,813091	,759918	7	6,23028	6,00205
,789409	,730690	8	7,01969	6,73274
,766416	,702587	9	7,78611	7,43533
,744094	,675564	10	8,53020	8,11090
,722421	,649581	11	9,25262	8,76048
,701380	,624597	12	9,95400	9,38507
,680951	,600574	13	10,63496	9,98565
,661118	,577475	14	11,29607	10,56312
,641862	,555264	15	11,93793	11,11839
,623167	,533908	16	12,56110	11,65230
,605016	,513373	17	13,16612	12,16567
,587394	,493628	18	13,75351	12,65930
,570286	,474642	19	14,32380	13,13394
,553676	,456387	20	14,87747	13,59033
,537550	,438833	21	15,41502	14,02916
,521892	,421955	22	15,93692	14,45112
,506692	,405726	23	16,44361	14,85684
,491934	,390121	24	16,93554	15,24696
,477606	,375117	25	17,41315	15,62208
,463695	,360689	26	17,87684	15,98277
,450189	,346817	27	18,32703	16,32959
,437077	,333477	28	18,76411	16,66306
,424346	,320651	29	19,18845	16,98371
,411987	,308319	30	19,60044	17,29203
,399987	,296460	31	20,00043	17,58849
,388337	,285058	32	20,38877	17,87355

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TABLE 1st. The Amount of one Pound for any Number of Years under 33, at ye Rates of 5 & 6 ⅌ Ct. ⅌ An: Com: Inter:	TABLE 2d. The Amount of 1£ Annuity for any number of Years under 33, at ye Rates of 5 & 6 ⅌ Ct. ⅌ an: Com: Interest.

Rates		Years	Rates	
5	6		5	6
1,05000	1,06000	1	1,00000	1.00000
1,10250	1,12360	2	2,05000	2.06000
1,15762	1,19101	3	3,15250	3.18360
1,21550	1,26247	4	4,31012	4.37461
1,27628	1.33822	5	5,52563	5.63709
1,34009	1,41852	6	6,80191	6.97532
1,40710	1.50363	7	8,14200	8.39383
1,47745	1.59384	8	9.54910	9.89746
1.55132	1,68948	9	11.02656	11,49131
1,62889	1.79084	10	12,57789	13,18079
1,71034	1,89829	11	14.20678	14.97164
1.79585	2.01219	12	15.91712	16.86994
1,88565	2,13292	13	17.71298	18.88213
1.97993	2,26090	14	19.59863	21.01506
2.07892	2,39655	15	21.57856	23.27597
2,18287	2,54035	16	23.65749	25.67252
2.29201	2,69277	17	25.84036	28.21288
2.40662	2.85434	18	28.13238	30.90565
2.52695	3.02559	19	30.53900	33.75999
2,65329	3,20713	20	33.06595	36.78559
2.78596	3.39956	21	35.71925	39.99272
2.92526	3.60353	22	38.50521	43.39229
3.07152	3,81975	23	41.43047	46.99582
3.22510	4,04893	24	44.50199	50.81557
3,38635	4.29187	25	47.72709	54.86451
3.55567	4.54938	26	51.11345	59.15638
3.73345	4.82234	27	54.66912	63.70576
3.92013	5.11168	28	58.40258	68.52811
4.11613	5.41838	29	62.34271	73.63979
4.32194	5.74349	30	66.43884	79.05818
4.53804	6.08810	31	70.76079	84.80167
4.76494	6.45338	32	75.29883	90.88977

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TABLE 3^{d} The present Worth of 1£ due at any number of years to come under 33. Rebate at 5&6 ℘C.^{t} ℘an: Com: Interest

TABLE 4th The present Worth of 1£ Annuity to continue any number of Years under 33. Rebate at 5&6 ℘C.^{t} ℘an: Com: Interest

Rates		years	Rates	
5	6		5	6
.952381	.943396	1	0.95238	0.94339
.907030	.889996	2	1.85941	1.83339
.863838	.839619	3	2.72324	2.67301
.822702	.792093	4	3.54595	3.46510
.783526	.747258	5	4.32947	4.21236
.746215	.704960	6	5.07569	4.91732
.710682	.665057	7	5.78637	5.58238
.676839	.627412	8	6.46321	6.20979
.644609	.591898	9	7.10782	6.80169
.613913	.558394	10	7.72173	7.36008
.584679	.526787	11	8.30641	7.88687
.556837	.496969	12	8.86325	8.38384
.530321	.468839	13	9.39357	8.85268
.505068	.442301	14	9.89864	9.29498
.481017	.417265	15	10.37965	9.71225
.458111	.393647	16	10.83777	10.10589
.436296	.371364	17	11.27406	10.47726
.415520	.350343	18	11.68958	10.82760
.395734	.330513	19	12.08532	11.15811
.376889	.311804	20	12.46221	11.46992
.358942	.294155	21	12.82115	11.76407
.341849	.277505	22	13.16300	12.04158
.325571	.261797	23	13.48857	12.30338
.310067	.246978	24	13.79864	12.55035
.295302	.232998	25	14.09394	12.78335
.281240	.219810	26	14.37518	13.00316
.267848	.207368	27	14.64303	13.21053
.255093	.195630	28	14.89812	13.40616
.242946	.184556	29	15.14107	13.59072
.231377	.174110	30	15.37245	13.76483
.220359	.164254	31	15.59281	13.92908
.209865	.154956	32	15.80267	14.08404

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and 6*l. per Cent. per Ann.* compound intereſt. Which table, as the others, will give the preſent worth of any ſum whatſoever, by only multiplying the value of 1*l.* by the ſum propoſed. Thus, ſuppoſe the foregoing example.

E X A M P L E.

3,54595 *the preſent worth of* 1l.*per an. for* 4*years.*
multiply'd by 50 *the annuity propoſed.*

gives 177,29750 *as before.*

The fourth table may be alſo form'd from the third, thus:

The firſt year is the ſame in both; the firſt and ſecond years of the third, added together, make the ſecond of the fourth; the ſecond year of the fourth, added to the third year of the third, makes the third of the fourth table, *&c.* See the example.

E X A M P L E, at 6l. *per Cent.*

,94339 * *The firſt year of both tables.*
,88999 *The ſecond year of the third table.*

1,83339 *The ſecond year of the fourth.*
,83961 *The third year of the third.*

2,673000 *The third year of the fourth.*
&c.

---

* *Note.* 1 is added to the right-hand figure, in conſideration of the places omitted.

More EXAMPLES *for the exerciſe of the foregoing* RULES and TABLES.

Ex. 1*ſt*. What will 1000*l.* increaſe to, forborn 22 years, at 5 *per Cent. per Annum* compound intereſt?

Anſwer, 2925*l.* 5*s.* 2*d.*½.

Ex. 2*d*. What would an annuity of 520*l.* forborn 17 years, at 6 *per Cent.* compound intereſt, amount to?

Anſwer, 14670*l.* 13*s.* 11*d.*¼.

Ex. 3*d*. What preſent money would diſcharge a debt of 7500*l.* to be paid at the end of ten years, rebate being made at 5*l. per Cent. per Annum*?

Anſwer, 4604*l.* 6*s.* 11*d.*¼.

Ex. 4*th*. What is the yearly rent of 65*l.* to continue 30 years, worth in ready money, rebate being made at 6 *per Cent. per Annum* compound intereſt?

Anſwer, 894*l.* 14*s.* 3*d.*¼.

Ex. 5*th*. What is the preſent worth of a reverſion of a leaſe of 500*l.* *per Annum*, to continue 20 years, but not to commence till after the end of five years, allowing the purchaſer 6 *per Cent. per Annum* compound intereſt?

Note, To work queſtions of this nature, there muſt be two diſtinct operations, *viz*

1*ſt*. The preſent worth of the propoſed annuity; for the given time of its continuance muſt be found as if it were immediately to commence. Then,

2*d*. See what principal, forborn at the given intereſt, would, in the time to the commencing of the reverſion, amount to the aforeſaid preſent worth, and that principal will be the preſent worth of ſuch annuity in reverſion.

Thus by table the 4th, the preſent worth of 1*l.* per Annum, *to continue* 20 *years, at* 6*l.* per Cent. *is found to be.*

11,46992

which multiplied by the propoſed annuity, 500

gives 5734,96000

Then, in the firſt table, againſt 5 years, the time to the commencing of the reverſion, under the ſame rate of intereſt, is found 1,33822, the amount of 1 *l.* in the ſaid time; therefore,

If 1,33822 *ariſe from* 1*l. from what principal does* 5734,96000 *ariſe?*

Anſwer, 4285,5135=4285*l.* 10*s.* 3*d.* ¼.

Or it may more eaſily be done by firſt finding the preſent worth of the annuity for the whole time, both of poſſeſſion and reverſion, and then deducting the preſent worth of the poſſeſſion; the remainder will be the preſent worth of the reverſion. Thus 12,78335, the tabular number againſt 25 years, the whole time being multiplied by 500, gives 6391,67500; from which ſubtracting 2106,18000, the produce of 4,21236 the tabular number againſt 5 years, the time of the poſſeſſion, the remainder 4285,49500, is the value of the reverſion; Within a trifle the ſame anſwer as before.

FREEHOLD ESTATES.

TO find the preſent worth of any annual rent, to continue for ever, commonly called fee-ſimple.

RULE.

Divide the propoſed rent by the intereſt of 1*l.* for one year, (which at 5*l. per Cent.* is ,05, or at 6 *per Cent.* ,06, as before ſhewn) and the quotient is the preſent value of the eſtate.

EXAMPLE.

What is an eſtate of 200*l. per Annum*, to continue for ever, worth in ready money, allowing the purchaſer 5*l. per Cent. per Annum*, compound intereſt?

—,05)200,00(
Anſwer, 4000*l.*

Or it may be done by multiplying the fee-ſimple of 1*l.* by the yearly rent propoſed.

The fee-ſimple of 1*l. per Annum*, compound intereſt, at

1 *per Cent.*	2 *per Cent.*	3 *per Cent.*	4 *per Cent.*	5 *per Cent.*
100,00000	50,0000	33,33333	25,00000	20,00000

6 *per Cent.*	7 *per Cent.*	8 *per Cent.*	9 *per Cent.*	10 *per Cent.*
16,66667*	14,28571	12,50000	11,11111	10,00000.

**Note*, In contracting a line of decimals from many to fewer places, 1 is added to the laſt figure retained, if the next omitted figure exceedeth 5; which rule hath been all along obſerved in the tables, &c.

The foregoing queſtion worked by the laſt rule.

	l.
The fee-ſimple *of* 1 l. *per Ann. at* 5 l. *per Cent. is*	20
which, multiplied by the yearly rent,	200
gives, as before,	4000

I will now only add another example or two, for the reader's exerciſe, and conclude this rule.

1. *A.* has the poſſeſſion of an eſtate of 130*l.* *per Annum*, to continue 20 years; *B.* has the reverſion of the ſame, from that time for ever. What muſt *A.* give *B.* if he would purchaſe his reverſion? And what muſt *B.* give *A.* if he would buy his poſſeſſion, accounting 6 *per Cent.* compound intereſt in each caſe?

Anſwer, { A'*s poſſeſſion is worth* 1491 . 1 . 9$\frac{1}{4}$
{ B'*s reverſion is worth* 675 . 11 . 6$\frac{1}{2}$

2. *A.* agrees with *B.* for an annuity of 600*l.* *per Annum*, to continue 25 years, to give him the preſent worth of it, at 6*l.* *per Cent.* *per Annum*; but not having money enough by him, offers to make over to him a freehold eſtate of 12 *l.* *per Annum*, at the ſame intereſt: What money beſides will pay his purchaſe?

Anſwer, 7470 *l.* 0*s.* 2*d.* $\frac{1}{2}$.

CHAP. VI.

Of FELLOWSHIP.

WITHOUT taking any notice of the common way of working this rule, obſerve this much more compendious method.

RULE.

Divide the whole gain, or loſs, by the whole ſtock, and multiply the quotient by each man's particular ſtock; the ſeveral products are the reſpective gains of each.

Note, It is neceſſary to place ſo many cyphers on the right hand of your dividend, as will bring out ſix or ſeven fractional places in the quotient: but if the laſt, or any figure of it will recur infinitely, put down the vulgar fraction in that place; and, to make up the ſeveral products, do as ſhewn in the indirect Rule of Three.

EXAMPLE.

Suppoſe *A*. *B*. and *C*. trading together, *A* puts in 500*l*. *B*. 700*l*. and *C*. 1200*l*. the whole gain is 836*l*. What ſhare of it belongs to each?

$$\left.\begin{array}{l} A = 500 \\ B = 7\,00 \\ C = 1200 \end{array}\right\} = 2400)836{,}0000(.\,348\tfrac{3}{9} = .\,348\tfrac{1}{3}$$

$$\begin{array}{r} .\,348\tfrac{1}{3} \\ 500 \\ \hline \end{array} \qquad \begin{array}{r} .\,348\tfrac{1}{3} \\ 700 \\ \hline \end{array} \qquad \begin{array}{r} .\,348\tfrac{1}{3} \\ 1200 \\ \hline \end{array}$$

$$\left.\begin{array}{l} A\text{'s share} = 174\;1\tfrac{2}{3} \\ B\text{'s} \ldots = 243.8\tfrac{1}{3} \\ C\text{'s} \ldots = 418.0 \end{array}\right\} \begin{array}{l} B\text{'s} = 243.8\tfrac{1}{3} \quad C\text{'s} = 418.0 \\ = 836l.\ \text{the total gain.} \end{array}$$

See another EXAMPLE *with time.*

A. puts into company 525*l.* 10*s.* for ſix months. *B.* 382*l.* 15*s.* for eight months; and *C.* 1000*l.* for four months; they gain in all 1286*l.* 12*s.* How much is that for each?

	525,5	382,75	1000
	6	8	4
A's *ſtock*	3153,0	3062,00	4000
B's ——	3062		
C's ——	4000		

10215)1286,600000(,125952

	,125952	,125952	,125952
	3153	3062	4000
A's *gain*	397,126656	385,665024	503,808000
B's ——	385,665024		
C's ——	503,808000		

Total, 1286,599680, *or* 1286*l.* 12*s.* *nearly.*

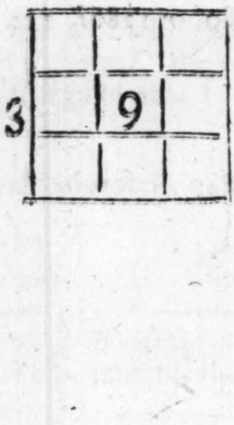
CHAP. VII.

Of EVOLUTION, *or* EXTRACTION *of* ROOTS.

The SQUARE ROOT.

EXTRACTION of the SQUARE ROOT is finding the ſide of a ſquare figure; or, numerically ſpeaking, it is finding what number, multiplied by itſelf, will produce the number given. Thus the ſquare root of 16 is 4, 4 times 4 making 16.

What a ſquare is, may be ſeen in the following figure: which being divided every way into 3 equal parts, its whole content or ſquare is 9, and its ſide or root is 3.



Square numbers are either ſingle or compound.

A ſingle ſquare number is always leſs than 100, being produced by the multiplication of ſome one ſingle figure by itſelf; as 16 from 4, *&c.* So that the root of any ſingle ſquare may be found in the annexed table. always taking the root of the next leſs ſquare, for any number not there inſerted; as, for 26 take 5, or 3 for 10, *&c.*

Square	1	4	9	16	25	36	49	64	81
Roots	1	2	3	4	5	6	7	8	9

A compound ſquare number being compoſed by the multiplication of two or more figures by themſelves, always exceeds 100; as 144, which is 12 times 12; or 225, which is 15 times 15, *&c.*

If the root therefore is expreſs'd by two figures, its ſquare muſt at leaſt conſiſt of three; for the leaſt root expreſs'd by two figures is 10, whoſe ſquare is 100. And if the root has three figures, its ſquare muſt at leaſt have five. If four, the ſquare has ſeven, *&c.* So that you cannot augment the root one figure, but you increaſe the ſquare two.

To find the Root of any compound ſquare Number; as ſuppoſe 2704.

1*ſt.* You muſt diſtinguiſh it into ſingle ſquares, by placing dots over every other figure, beginning at the right-hand thus;

$$2\dot{7}0\dot{4}$$

And ſo many dots as happen, ſo many places will the root conſiſt of; which in this example are two.

2*dly*. Drawing a crooked line on the right-hand of your number, as in diviſion, find the root of your firſt ſingle ſquare, and place it in the quotient.

See the work.

· ·
2704(5

3*dly*. Placing the ſquare of the root found, under the firſt ſingle ſquare, ſubtract, and ſet down the remainder; bringing down to it the next ſingle ſquare, call the line a *Reſolvend*.

· ·
2704(5
25
. 204 *Reſolvend*.

4*thly*. Drawing another crooked line on the left-hand of the ſaid reſolvend, place beyond it the double of the quotient, in form of a diviſor,

· ·
2704(5
25
10).204

5*thly*. Dividing the reſolvend, all but the unit's place, by the ſaid diviſor (which, in this example, is the 10's in 20) ſet down the number of times it goes (*viz*. 2) both in the quotient, and on the right-hand of the diviſor.

· ·
2704(52
25
102)204 *Reſolvend*.

6thly. Multiplying the whole diviſor by the figure laſt placed in the quotient, ſet down the product under and ſubtract it from the reſolvend.

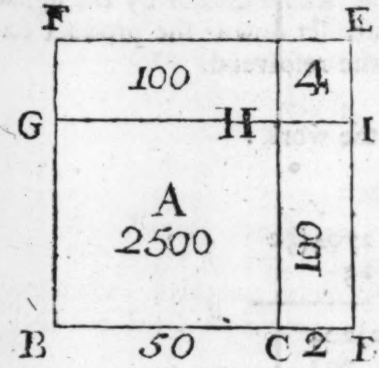
See the work:

```
 .  .
2704(52
25
-------
102)204
    204
   ----
     .0
```

Note, If the diviſor multiply'd by the quotient-figure, gives a product greater than the reſolvend, 'tis falſe, and muſt be rectified by a ſmaller quotient-figure.

Extraction of the ſquare root being a matter rather lineal than numeral, the reaſon of the rules above given for its operation, will beſt appear from the following figure: Wherein let it be conſidered, that if the root *B. C.* of the ſquare *A,* be augmented by the magnitude *C. D.* the ſaid ſquare will be inlarged by the figure *C. D. E. F. G. H. I.* equal to an oblong or long ſquare, whoſe length ſhall be the root *B C.* twice taken, with the line *C. D.* and breadth the ſame line *C. D.*

N. B. The particular contents of the ſeveral parts of the figure are as therein ſet down.



The ſeveral parts of the Square,	2500 100 100 4
Whole ſquare,	2704
Parts of the Root,	50 2
Whole Root,	52

In the operation therefore, having taken the neareſt root of the firſt ſquare 27, *viz.* 5, and placed it in the quotient, from the place it ſtands in (*viz.* the ſecond) it muſt be accounted 50; the ſquare of which, *viz.* 2500 (marked *A.* in the figure) being ſubtracted from the given number 2704, there remains 204, equal to the figure *C. D. E. F. G. H. I* In order to find the content of which, I double the root *B. C. viz.* 50, for the two baſes *C. H.* and *H. G.* which making 100, I ſet it down as a diviſor; and to find the height of the ſaid figure, *viz. C. D.* I ſeek how often the ſaid diviſor, is contain'd in the number 204; and finding it twice, I ſet down two in the quotient for the ſaid height and alſo in the unit's place of my diviſor for the baſe *H. I.* Then multiplying the baſes by the height, I have the content of the whole annexed figure *C. D. E. F. G. H. I. viz.* 204, exactly equal to the number remaining, after the ſubtraction of the firſt great ſquare 2500; ſo that the number given, *viz.* 2704, appears to be a perfect ſquare, whoſe root or ſide is exactly 52.

Note, As many ſingle ſquares as are in the number propoſed, ſo many times muſt the work of the three laſt rules be repeated; every ſquare being to be brought down, as directed in the third rule.

EXAM-

EXAMPLE.

· · ·

What's the root of 582169(763 *Anſwer.*

49

146).921 *Reſolvend.*

876

1523).4569 *Reſolvend.*

4569

0

In this example, to 45 the ſecond remainder, 69 the next ſquare, being brought down, makes 4569 for a new reſolvend; and 76, the quotient-figures, being doubled for a new diviſor, makes 152, which going three times in the reſolvend, 3 is placed both in the quotient, and on the right-hand of the diviſor, which being then multiplied by 3, gives 4569, equal to the reſolvend, ſo that nothing remains.

PROOF.

The Proof of this extraction, is only multiplying the root found by itſelf; which if right will produce the ſquare number given. *Note,* if any thing remains, it muſt be taken in.

To find the fractional Part of the Root.

Add to the number given a competent number of pairs of cyphers, and proceed by the foregoing rules: As ſuppoſe 75896 were given for an example.

EXAMPLE.

```
            .  .  .
            75896,000000(275,492
            4
            ----
       47)358
          329
          ----
     545).2996
          2725
          -----
     5504).27100
           22016
           -----
    55089).508400
            495801
            ------
   550982).1259900
            1101964
            -------
            .157936
```

Note, As many pairs of cyphers as are added, ſo many fraction places of decimal parts will be in the root.

To extract the SQUARE ROOT *of a* VULGAR FRACTION,

FIND the root of the numerator for a numerator; and let the root of the denominator be its denominator; thus the ſquare root of $\frac{16}{25}$ is $\frac{4}{5}$, and of $\frac{4}{9}$ is $\frac{2}{3}$, *&c.*

The root of a mixed number is alſo found after the ſame manner, being firſt reduced into an improper fraction; thus $6\frac{1}{4}$, reduced, makes $\frac{25}{4}$, the root of which is $\frac{5}{2}$, or $2\frac{1}{2}$.

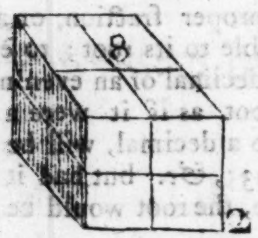
But if either a proper fraction, or a mixed number, be incommenſurable to its root; to extract it, reduce the fraction to a decimal of an even number of places, and extract the root as if it were a whole number: thus $\frac{7}{8}$, reduced to a decimal, will be ,8750, the ſquare root of which is ,935, *&c.* but had it been $4\frac{7}{8}$, which reduced is 4,8750, the root would be 2,20794, *&c.*

The CUBE ROOT.

EXtraction of the CUBE ROOT is finding the ſide of a ſolid figure, whoſe length, breadth, and depth are equal; or numerically ſpeaking, it is finding what number multiply'd into itſelf, and then into the product, will produce the number given; thus the Cube Root of 64 is 4, 4 times 4 being 16, and 4 times 16 make 64.

What a Cube is may be ſomething explain'd by the following figure; wherein 8 dice are ſo diſpoſed, that there are two every way, that is two in length, 2 in breadth, and 2 in height; ſo that 8 is the cube, and 2 the root.

See the figure.



Cube numbers are either ſingle or compound.

A ſingle-cube number is always leſs than 1000, being produced by the multiplication of one ſingle figure, firſt by itſelf, and then by its product; as 125 from 5, &c. So that the root of any ſingle cube may be found in the annexed table, always remembring to take the root of the next leſs cube for any number not there inſerted, as for 513 take 8, or 6 for 220, &c.

Cubes	1	8	27	64	125	216	343	512	729
Squrs.	1	4	9	16	25	36	49	64	81
Roots.	1	2	3	4	5	6	7	8	9

A compound cube-number, being compoſed by the multiplication of 2 or more figures, firſt by themſelves, and then by their product, always exceeds 1000, as 1728 from 12, or 15625 from 25, &c.

If therefore the root is expreſs'd by two figures, its cube muſt at leaſt conſiſt of four; for the leaſt root expreſſed by two figures is 10, whoſe cube is 1000; if the root has three figures, its cube muſt at leaſt have ſeven, &c. So that you cannot augment the root one figure, but you increaſe the cube three.

Therefore, to find the root of any compound cube-number, as ſuppoſe 15625.

1*ſt*. You muſt diſtinguiſh it into ſingle cubes, by placing dots over every third figure, beginning at the right-hand, thus :

$$\dot{1}\dot{5}62\dot{5}$$

And ſo many dots as happen, ſo many places will the root conſiſt of ; which, in this example, are two.

2*dly*, Drawing a crooked line on the right-hand of your number, as in diviſion, ſet down as a quotient the root of your firſt ſingle cube ; which in this example, is 2.

3*dly*, Placing 8 the cube of 2, the root found, under 15 the firſt ſingle cube, ſubtract, and to the remainder 7 bring down 625, the next ſingle cube, and it will make 7625 ; which call a Reſolvend.

See the work.

$$\begin{array}{l} 1\dot{5}62\dot{5}(2 \\ \underline{\ \ 8\ \ \ \ } \\ \ \ 7625\ \textit{Reſolvend.} \end{array}$$

4*thly*, Draw a line under the reſolvend, and tripling the ſquare of the root 2, ſet the ſaid triple ſquare (*viz.* 12) under the reſolvend ; ſo that units in the ſaid triple ſquare may ſtand under the place of hundreds in the reſolvend.

5*thly*, Subſcribe alſo the triple of the root 2, (*viz.*6) ſo that units, in this, may ſtand under the place of tens in the reſolvend.

6*thly*. The triple-ſquare of the root, and triple root being placed as directed, draw a line under them, and add them together in the order they are placed; the ſum 126 is a diviſor.

7*thly*. Accounting all the reſolvend, except the place of units, (*viz.* 762) a dividend, ſeek how often the diviſor (126) is contained in it, and place the number of times (which is here 5) in the quotient.

8*thly*. Draw a line under the diviſor, and multiplying the triple ſquare (12) by 5, the figure laſt placed in the quotient, ſet the product (60) ſo under the ſaid triple ſquare, that units may ſtand under units, and tens under tens.

9*thly*. Squaring the figure (5) laſt placed in the quotient, multiply its ſquare (*viz.* 25) by the triple root (6), and place the product (150) ſo, that units, in this may ſtand under units in the ſaid triple number.

10*thly*. Subſcribe the cube of (5), the figure laſt placed in the quotient, which will be 125; ſo that tens, in this, may ſtand under units in the former product.

11*thly*. Then drawing a line, add the three numbers laſt placed together; and ſubtracting the ſum 7625 (which is called the *ablatitium*) from the reſolvend, ſet down the remainder (if any) in order underneath, as in common ſubtraction.

See the work.

15625(25
8 Cube of 2.

7625 Reſolvend.

12 triple ſquare of 2.
6 triple of the Root 2.

126 Diviſor.

60 triple ſquare × 5.
150 triple Root × 5 × 5.
125 Cube of 5.

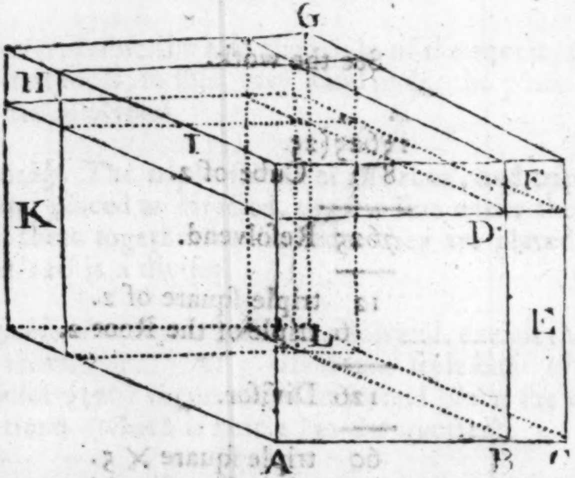
7625 Ablatitium.

...0

Note, If the given number hath more places, the next ſingle cube muſt be brought down to the laſt remainder, for a new reſolvend; and the work of the 4th, 5th, 6th, 7th, 8th, 9th, 10th, and 11th rules, muſt be repeated as often as you ſo form a new reſolvend.

Note alſo, if the *ablatitium* is greater than the reſolvend, the work is falſe, and muſt be rectified by placing a leſſer figure in the quotient.

To account for theſe rules of extracting the cube root, we muſt again make uſe a figure with literal references, as follows:



From whence it appears, That if the root *A. B.* of the cube *A. D.* be augmented by the magnitude *B. C.* the cube is thereby enlarged by the addition of ſeven parallelepipedes;[*] *viz.* 1*ſt.* The little cube *G*, whoſe root is the line *B. C.* 2*d.* The three parallelepipedes, mark'd *I. K. E.* which have the ſquare of the root *A. B.* for baſe, and the line *B. C* for height. 3*d.* The three parallelepipedes mark'd *F. H. L.* which have the ſquare of the line *B.C.* for baſe, and the root *A. B.* for height.

In the operation therefore of the foregoing example, 15625 being divided into ſingle cubes, it appears that the root will conſiſt of two places: The neareſt root then of the firſt cube 15, *viz.* 2, being placed in the quotient where its value will be 20, from the place it ſtands in, and its cube 8000 ſubtracted from the given number 15625, the remainder 7625, will be equal to the ſeven parallelepipedes, *G. I. K. E. F. H. L.* the baſes of three of which, *viz. I. K. E.* being the ſquare of the root *A. B.* three times the ſquare of 20, *viz.*

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[*] *Parallelepipedes is a geometrical term, ſignifying regular figures, whoſe oppoſite ſides and angles are equal.*

viz. 1200 is ſet down under the ſaid remainder, or reſolvend; alſo three more of the ſaid parallelepipedes, *viz. F. H. L.* having their height equal to the ſaid root three times 20, *viz.* 60, the ſum of the three heights, is added to the ſaid 1200, the ſum of the baſes of the other three, for a diviſor; by which dividing the reſolvend, the number of times it goes, *viz.* 5, is the height of the three parallelepipedes *I. K. E.* and the root of the ſquare of the baſes of the other three parallelepipedes *F. H. L.* and alſo the root or ſide of the parallelepipedon, or little cube *G.* Multiply therefore 1200 by 5, for the whole contents of the parallelepipedes *I.K.E. viz.* 6000; and ſquaring the ſaid 5 for the baſes of the three parallelepipedes *F. H. L.* multiply their three heights 60, by the ſaid ſquare of their baſes, the product is 1500, their whole contents; under which ſet down alſo the cube of 5, *viz.* 125, for the content of the parallelepipedon *G.* the ſum of theſe three numbers I find to be juſt 7652, exactly equal to the reſolvend; which ſhews the number given to be a perfect cube, whoſe root is exactly 25.

To make all more plain, ſee the work at length, with the vacant places fill'd with cyphers.

15625 ———— **20**

8000 *Cube* A. D. — 5

7625 ***Remainder, or Reſolvend.*** — **25** ***Root.***

1200 *Baſes of Parallelepipedes* I. K. E.

60 *Height of Parallelepipedes* F. H. L.

1260 *Diviſor.*

6000 *Content of Parallelepipedes* I. K. E.

1500 *Content of Parallelepipedes* F. H. L.

125 *Content of Parallelepipedon* G.

7625 *Total, or Ablatitium.*

...,0

PROOF.

PROOF.

The proof of this extraction is, by multiplying the root found, firſt by itſelf, and that product again by the root; adding the remainder, if there be any, to the laſt product: thus,

25 *the Root.*
multiply'd by 25

125
50

gives 625
again by 25 *the Root.*

3125
1250

gives 15625

To find the fractional part of the Root.

Add to the number given, ſo many ternaries of cyphers, as you would have places of decimal parts in the root, and proceed by the foregoing rules. As ſuppoſe 865635 were given for an example.

EXAM-

EXAMPLE.

865	635	000	000	000	(95,304 *Root.*
729					
136	635				1ſt *Reſolvend.*
24	3				
	27				
24	57				*Diviſor.*
121	5				
6	75				
	125				
128	375				*Ablatitium.*
8	260	000			2d *Reſolvend.*
2	707	5			
	2	85			
2	710	35			*Diviſor.*
8	122	5			
	25	65			
		27			
8	148	177			*Ablatitium.*
	111	823	000		3d *Reſolvend.*
	272	462	7		
		28	59		
	272	491	29		*Diviſor.*
	111	823	000	000	4th *Reſolvend.*
	27	246	270	0	
			285	90	
	27	246	555	90	*Diviſor.*
	108	985	080	0	
		4	574	40	
				64	
	108	989	654	464	*Ablatitium.*
	.2	833	345	536	*Remainder.*

In this example, there being three ternaries of cyphers, the root muſt have three fractional places.

Note, The diviſor going no times in the third reſolvend, the ſame figures are immediately brought down again, and with a new ternary of cyphers form the 4th reſolvend.

To extract the Cube Root of a vulgar Fraction.

Find the root of the numerator given for a new numerator, and the root of the denominator given for a new denominator; thus the cube root of $\frac{64}{125}$ is $\frac{4}{5}$.

But if the vulgar fraction be incommenſurable to its root, reduce it to a decimal of three, ſix, nine, or any ternary of places, and then extract its root.

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